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Mathematics in Engineering and Technology

Er. R.K. Rajesh
(DIRECTOR)

As rightly said by great mathematician Rene' Descartes ***"With me, everything turns into mathematics"***. The GATE conducting body has incorporated ***the feel of Descartes*** into its exam system and made mathematics inevitable for the students of all the streams. Hence knowledge of Mathematics is very important for engineering graduates to score good marks into Graduate Aptitude Test in Engineering (GATE) and many reputed examinations. The importance of mathematics has never been greater than now and for the foreseeable future. Mathematical skills are crucial for a wide array of analytical, technological, scientific, security and economic applications. A deficiency in certain basic mathematics skills dramatically limits your opportunities to crack the examinations.

Engineering Mathematics, like any other subjects, is important to the extent to which it supports and contributes to the purpose of engineering education. In engineering field there are enormous uses of mathematics to calculate various factors in the field of Computer science, Civil, Chemical, Electrical, Electronics, Mechanical, Aeronautical, Instrumentation engineering etc.

Preparation for Engineering Mathematics can't be underestimate for GATE examination as it will carry around **15% of the total marks** for AE, AG, BT, CE, CH, CS, EC, EE, IN, ME, MN, MT, PI, TF and XE paper.

GATE examinations rigorously tests candidates' overall knowledge & understanding of concepts, ability to apply their knowledge and personality level by screening them through basics knowledge of subjects. The syllabus and questioning pattern has remained pretty much the same over the years. This book has been composed with an aim to provide examination oriented theories followed y solved examples for better understanding of concepts. It consists of huge range of replica questions for practice purpose to assure good preparation level.

This book has been drafted by our team who are experts from IITs and having good analysis of previous year papers. This book will provide examination oriented theories followed b y solved examples for better understanding of concepts. It consists of huge range of replica questions for practice purpose to assure good preparation level.

Please write me if you have any comments, suggestions or corrections to share.

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ENGINEERING MATHEMATICS
GATE Syllabus

CIVIL ENGINEERING – CE
CHEMICAL ENGINEERING – CH
MECHANICAL ENGINEERING – ME

Linear Algebra: Matrix algebra, Systems of linear equations, Eigen values and eigenvectors.

Calculus: Functions of single variable, Limit, continuity and differentiability, Mean value theorems, Evaluation of definite and improper integrals, Partial derivatives, Total derivative, Maxima and minima, Gradient, Divergence and Curl, Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green's theorems.

Differential equations: First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Cauchy's and Euler's equations, Initial and boundary value problems, Laplace transforms, Solutions of one dimensional heat and wave equations and Laplace equation.

Complex variables: Analytic functions, Cauchy's integral theorem, Taylor and Laurent series.

Probability and Statistics: Definitions of probability and sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Poisson, Normal and Binomial distributions.

Numerical Methods: Numerical solutions of linear and non-linear algebraic equations Integration by trapezoidal and Simpson's rule, single and multi-step methods for differential equations.

COMPUTER SCIENCE & INFORMATION TECHNOLOGY – CS

Mathematical Logic: Propositional Logic; First Order Logic.

Probability: Conditional Probability; Mean, Median, Mode and Standard Deviation; Random Variables; Distributions; uniform, normal, exponential, Poisson, Binomial.

Set Theory & Algebra: Sets; Relations; Functions; Groups; Partial Orders; Lattice; Boolean Algebra.

Combinatorics: Permutations; Combinations; Counting; Summation; generating functions; recurrence relations; asymptotics.

Graph Theory: Connectivity; spanning trees; Cut vertices & edges; covering; matching; independent sets; Colouring; Planarity; Isomorphism.

Linear Algebra: Algebra of matrices, determinants, systems of linear equations, Eigen values and Eigen vectors.

Numerical Methods: LU decomposition for systems of linear equations; numerical solutions of non-linear algebraic equations by Secant, Bisection and Newton-Raphson Methods; Numerical integration by trapezoidal and Simpson's rules.

Calculus: Limit, Continuity & differentiability, Mean value Theorems, Theorems of integral calculus, evaluation of definite & improper integrals, Partial derivatives, Total derivatives, maxima & minima.

ELECTRICAL ENGINEERING – EE
ELECTRONICS AND COMMUNICATION ENGINEERING – ECE
INSTRUMENTATION ENGINEERING-IN

Linear Algebra: Matrix Algebra, Systems of linear equations, Eigen values and eigen vectors.

Calculus: Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series. Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green's theorems.

Differential equations: First order equation (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's and Euler's equations, Initial and boundary value problems, Partial Differential Equations and variable separable method.

Complex variables: Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent' series, Residue theorem, solution integrals.

Probability and Statistics: Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, Normal and Binomial distribution, Correlation and regression analysis.

Numerical Methods: Solutions of non-linear algebraic equations, single and multi-step methods for differential equations.

Transform Theory: Fourier transform, Laplace transform, Z-transform.

ENGINEERING MATHEMATICS CONTENTS

CHAPTER-1

MATRICES

Important Terms Related to Matrices:

(*i*) Element of Matrix: Each of the *mn* numbers of *m* $m \times n$ matrix is called an element.

(*ii*) Leading Element: The element lying in the first row and first column is called leading element of a matrix.

(*iii*) Diagonal Elements: An element of a matrix A = $[a_{ij}]$ is said to be diagonal element if $i = j$. Thus an element where row suffix equal to the column suffix is a diagonal element a_{11}, a_{22}, a_{33}. Are all diagonal elements.

(*iv*) Principal Diagonal: The line along which the diagonal elements lie is called the principal diagonal or simply the diagonal of the matrix.

Type of Matrix : (*i*) Row Matrix: The matrix having order $1 \times n$ or matrix having only one row is called row matrix in row matrix. The number of columns may be '*n*' where $n \in \mathrm{N}$.

(*ii*) Column matrix: The matrix having order $m \times 1$ or matrix having only one column is called column matrix in column matrix, the no. of rows may be '*n*' where $n \in \mathrm{N}$.

(*iii*) Zero matrix: A matrix each of where elements is zero is called zero matrix or null matrix. A zero matrix of order $m \times n$ is denoted by $0_{(\mathrm{m} \times \mathrm{n})}$

(*iv*) Square Matrix: A matrix in which the number rows is equals to the number of columns is called a square matrix. Otherwise it is said to a rectangular matrix. Thus a matrix $\mathrm{A} = [a_{ij}]_{m \times n}$ is said to be a square matrix. If $m = n$ and a rectangular matrix if $m \neq n$.

(*v*) Diagonal matrix: A square matrix $\mathrm{A} = [a_{ij}]$ is said to be a diagonal matrix. If all its non-diagonal elements are zero.

(*vi*) Unit matrix or identity matrix: $\mathrm{A} = [a_{ij}]_{n \times n}$ is said to be a unit matrix or identity matrix.

$$a_{ij} = \begin{cases} 0 & \text{when } i \neq j \\ 1 & \text{when } i = j \end{cases}$$

(*vii*) Scalar matrix: $\mathrm{A} = [a_{ij}]_{n \times n}$ is a scalar matrix if any $a_{ij} = \begin{cases} 0 & \text{when } i \neq j \\ k & \text{when } i = j \end{cases}$

(*viii*) Upper Triangular matrix: $\mathrm{A} = [a_{ij}]_{n \times n}$ is an upper triangular matrix. If $a_{ij} = 0$ for $i > j$

(*ix*) Triangular matrix: A matrix which is either a lower triangular matrix or an upper triangular matrix is called a triangular matrix.

(*x*) Lower Triangular matrix: $\mathrm{A} = [a_{ij}]_{n \times n}$ is a lower triangular matrix $a_{ij} = 0$ for $i < j$.

(*xi*) Triple Diagonal matrix: A square matrix is triple diagonal matrix. If all of its elements except on principal diagonal and the diagonal lying above the below it are zero.

(*xii*) Trace: The sum of all diagonal elements of matrix is called trace. This is defined only for a square matrix. Trace = Σa_{ij} when $i = j$

(*xiii*) Comparable matrix: Two matrix are said to be comparable. When they are of the same type. Thus two matrices. $\mathrm{A} = [a_{ij}]_{m \times n}$ and $\mathrm{B} = [b_{ij}]_{p \times q}$. Are comparable. If $m = p$ and $n = q$.

(*xiv*) Equality of two matrix: Two matrix A and B are said to be equal (written as A = B). If

(*a*) They are of the same type and

(*b*) Their corresponding elements are equal.

(*xv*) Symmetric matrix: A matrix A is said to be symmetric. If $A' = A$. If the transpose of a matrix is equal to itself.

(*xvi*) Skew symmetric matrix: A matrix A is said to be skew symmetric. If $A' = -A$. When A matrix equal the negative of its transpose.

(*xvii*) Idempotent matrix: A square matrix A is called idempotent provided. It satisfies the relation $A^2 = A$.

(*xviii*) Periodic Matrix: A square matrix A is called periodic. If A^{k+1} = A where *k* is a positive integer. If *k* is the least positive integer for which $A^{k+1} = A$. Then *k* is said to be period of A.

(*xix*) Nilpotent matrix: A square matrix A is called nilpotent matrix of order *k* provided. It satisfies the relation $A^k = 0$ and $A^{k-1} \neq 0$ where *k* is order of nilpotent matrix A.

(*xx*) Involutary matrix: A square matrix A is called involuntary provided.

It satisfies the relation $A^2 = I$ where I is identity matrix.

(*xxi*) Orthogonal matrix: A square matrix A is called an orthogonal matrix. If the product of the matrix A and its transpose A' is an identity matrix $AA' = I$.

Multiplication of a Matrix by a Scalar:

Let $A = [a_{ij}]_{m \times n}$

$$\alpha A = [a_{ij}] = [\alpha\, a_{ij}]_{m \times n}$$

(*a*) Properties of Multiplication by a Scalar:

If $A = [a_{ij}]$

$B = [b_{ij}]$ are matrices of the same type α and β.

Are my scales

$$\alpha(A + B) = \alpha A + \alpha B$$

$$(\alpha + \beta) A = \alpha A + \beta A$$

$$\alpha(\beta A) = (\alpha\beta) A$$

Multiplication of Matrix: Let $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{n \times p}$ be two matrices conformable for the product AB. Then AB is defined as the matrix.

$$C = [c_{ij}]_{m \times p}$$

where,
$$C_{ij} = \sum_{k=1}^{n} a_{ik}\, b_{kj} \quad (i = 1, 2, ... m)$$

$$(j = 1, 2, ... p)$$

Properties of Matrix Multiplication:

(*i*) Matrix multiplication is associative.

(*ii*) Multiplication of matrices is distributive with respect to the addition of matrix.

Transpose of a Matrix: Given a matrix A, then the matrix obtained from A by changing the row's into columns and columns into rows is called the transpose of A and is denoted by A' and $\overline{A}$.

Properties of Transpose of matrix: (*i*) $(A')' = A$

(*ii*) $(A + B)' = A' + B'$

(*iii*) $(k\,A)' = k\,A'$ where *k* is any scales

(*iv*) $(AB)' = B'\,A'$

in general $(A_1A_2 \ldots\ldots A_{n-1}A_n)' = A'_n\ A'_{n-1} \ldots\ldots A'_2\ A'_1$

(*v*) If A is an invertible matrix then $(A^{-1})' = (A')^{-1}$

Adjoint of square matrix: The adjoint of a square matrix is the transpose of the matrix obtained by replacing each element of A by its co factor in |A|. If $A = [a_{ij}]_{n \times n}$. Then adjoint of a briefly written as Adj(A), adjoint A = $[A_{ij}]$ where A_{ij} is the co-factor of a_{ij} in |A|.

Properties of Adjoint:

(*i*) Adj(0) = 0
(*ii*) Adj(scaler) = Scaler
(*iii*) Adj(1) = I
(*iv*) Adj(diagonal) = (diagonal)
(*v*) $\text{Adj}(A^{-T}) = (\text{Adj } A)^{-T}$
(*vi*) A is symmetric from Adj(A) is symmetric
(*vii*) $\text{Adj}(\lambda A) = \lambda^{n-1}\,\text{Adj}A$ where *n* is order of matrix A.
(*viii*) adj(AB) = (adj B) (adj A)
(*ix*) If A is skew symmetric matrix of order '*n*'. Then adj A is
(*a*) Skew symmetric when *n* is even
(*b*) Symmetric when *n* is odd.
(*x*) A(adj A) = (adj A)A = $|A|\ I_n$ where I_n is unit matrix of order *n*.
(*xi*) |Adj A| = $|A|^{n-1}$
(*xii*) adj(adj A) = $|A|^{n-2}\ A\ I_n$ (A is non-singular matrix)
(*xiii*) |Adj (adj A)| = $|A|^{(n-1)^2}$ (A is non-singular matrix)
(*xiv*) A and Adj A behave alike
(*a*) If A is singular. Then adjoint of A is singular
(*b*) If A is non singular then adjoint of A is non singular
(*c*) If A is invertible. Then adjoint of A is also invertible.

A. Inverse of a Matrix: Let A be an *n*-rowed square matrix. If there exists in *n*-rowed square matrix B matrix that

$$AB = BA = I_n$$

Then the matrix A is said to be invertible and B is called the inverse of A or reciprocal of A.

B. Finding the inverse of matrix using adjoint matrix:

$$A\,.\,(\text{Adj } A) = |A|\,I$$

or
$$\frac{A\,.\,(\text{Adj } A)}{|A|} = I \text{ provided } |A| \neq 0$$

Rule for Solving System of equations by using matrix method:

(*a*) When system of equations is non homogenous

(*i*) If $|A| \neq 0$ then the system of equation is consistent and has a **Unique solutions** given by

$$X = A^{-1} B$$

(*ii*) If $|A| = 0$ and $(\text{Adj } A) B \neq 0$. Then the system of equations is inconsistent and has **no solution.**

(*iii*) If $|A| = 0$ and $(\text{Adj } A) B = 0$. Then the system of equations is consistent and has an **Infinite number of solutions.**

(*b*) When System of equations is Homogeneous:

(*i*) If $|A| \neq 0$ the system of equations have only trival solutions and it has **one solution.**

(*ii*) If $|A| \neq 0$ the system of equation has non trivial solutions and its has **infinite solutions.**

(*iii*) If number of equations < Number of unknowns, then it has **non-trivial solution.**

Rank of a Matrix: According to fundamental theorem of linear algebra. The dimension of the column space of a matrix equal the dimensions of the row space and the column value is called rank of the matrix we denote the rank of the matrix A by rank A.

For any matrix A rank A = rank A'

$$\text{Rank } (A + B) \leq \text{Rank } A + \text{Rank } B$$

If singular matrix $|A| = 0$ is rank of matrix 2 $\rightarrow$ In 3×3 matrix

If non singular matrix $|A| \neq 0$ rank of matrix 3 $\rightarrow$ In 3×3 matrix

Definition: The matrix $(A - \lambda I)$ is called characteristic matrix of A where I is the unit matrix of order *n*. Also the determinant.

$$|A - \lambda I| = \begin{vmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ a_{n1} & a_{n2} & \dots & a_{nm-\lambda} \end{vmatrix}$$

$(A - \lambda I) = 0$ is called 'characteristic equations of A'

Note: (*i*) The roots of the characteristic equations are called "characteristic roots or characteristic values or latest roots or proper values or eigen values" of the matrix A. The set of eigen value of A is called the "spectrum of A".

(*ii*) If λ is a characteristic root of $n \times n$ matrix A. Then A non zero vector X such that $AX = \lambda X$ is called characteristic vector or eigen vector of A corresponding to characteristic root λ.

Some Results Regarding Characteristic roots and Characteristic vectors:

(*i*) λ is a characteristic root of a matrix A. If there exists a non-zero vector X such that $AX = \lambda X$.

(*ii*) If X is a characteristic vector of a matrix A corresponding to characteristic value λ then *kx* is also a characteristic vector of A corresponding to the some characteristic value λ where *k* is a non-zero vector.

(*iii*) If X is a characteristic vector of a matrix A. Then X cannot correspond to more than one characteristic values of A.

(*iv*) The characteristic roots of a Hermitrain matrix are real.

(*v*) The characteristic roots of a real symmetric matrix are all real sign every such matrix is Hermetian.

(*vi*) The characteristic roots of a unitary matrix are of unit modulas $|\lambda| = 1$.

(*vii*) The characteristic roots of an orthogonal matrix is also of unit modulas since every real matrix is unitary.

Properties of Eigen Value:

(*i*) If $\lambda_1, \lambda_2, \lambda_3, ...$ be the eigen vectors of A. Then $k\lambda_1, k\lambda_2, k\lambda_n$ are eigen values of *k*A.

(*ii*) The eigen values of A^{-1} are the reciprocals of eigen values of A. If $\lambda_1, \lambda_2, \lambda_3.........$ are two eigen values of A. Then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3} . \frac{1}{\lambda_n}$ are the eigen value of A^{-1}.

(*iii*) Eigen values of A = Eigen value of A^T

(*iv*) Maximum no. of distinct eigen values = Size of A.

(*v*) Sum of eigen values = Trace of A = Sume of diagonal elements.

(*vi*) Product of eigen values = | A |

(*vii*) In a triangular and diagonal matrix, eigen are diagonal elements themselves.

(*viii*) If $\lambda_1, \lambda_2, ...\lambda_n$ are the eigen values of a non singular matrix A. Then $\frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}, \frac{|A|}{\lambda_3} ...... \frac{|A|}{\lambda_n}$ are the eigen values of Adj A.

The Cayley Hamilton Theorem: Every square matrix satisfies its own characteristic equation.

$C_0\lambda^n + C_1\lambda^{n-1} + ... + C_{n-1}\lambda + C_n = 0$ is the characteristic equation of a square matrix A of order *n*. Then

$$C_0A^n + C_1A^{n-1} + ... + C_{n-1}A + C_n = 0$$

Where λ is replaced by A in the characteristic equation.

Diagonalizable of Matrix:

If an *n* × *n* matrix A has a basis of eigen vectors. Then

$$D = X^{-1}AX$$

is diagonal, with the eigen values of A. As the entires on the main diagonal. Here X is the matrix with these eigen vectors as column vectors also,

$$D^m = X^{-1} A^m X \qquad m = 2, 3 ....$$

Hermition, Skew Hermitian and Unitary matrix:

A square matrix A = $[a_{ij}]$ is called

Hermition if	$\overline{A}^T = A$	or	$A^\theta = A$
Skew Hermitian if	$\overline{A}^T = -A$	or	$A^\theta = -A$
Unitary if	$\overline{A}^T = A^{-1}$	or	$AA^\theta = 1$

LEVEL-1

1. How many solutions does the following system of linear system have?

$$2x + 3y + 5z = 23$$
$$x + 4y + 3z = 18$$
$$5x + 15y + 14z = 70$$

(*a*) Unique (*b*) Infinite (*c*) No solution (*d*) None of these

2. If A is a square matrix of order 3 and with determinant of $\text{Adj}(\text{Adj}(A^{-1}))$ equals

(*a*) 16 (*b*) $\frac{1}{16}$ (*c*) $\frac{1}{64}$ (*d*) $\frac{1}{256}$

3. If A and B are symmetric matrices of some orders them $\overline{(A+B)}$ is

(*a*) Always symmetric (*b*) Skew symmetric (*c*) Either (*a*) or (*b*) (*d*) Neither (*a*) or (*b*)

4. If A and B are two matrices of order 3 × 5 and 5 × 3 respectively then determinant of the matrix 4BA equals.

(*a*) $4\,|B|\,|A|$ (*b*) $4^3\,|B|\,|A|$ (*c*) $4^5\,|A|\,|B|$ (*d*) None of these

5. Determinant of an orthogonal matrix equals

(*a*) + 1 (*b*) – 1 (*c*) (*a*) or (*b*) both (*d*) None of these

6. Matrix multiplication is always

(*a*) Abelian (*b*) Associative (*c*) Non-Abelian (*d*) Both (*b*) or (*c*)

7. The characteristic roots of unitary matrix are

(*a*) 1 (*b*) ± 1 (*c*) Of unit modulas (*d*) None

8. If Eigen value of a 3 × 3 matrix whose determinant equals 12 is 1, 2, and 3 then eigen values of adjoint of that matrix will be

(*a*) 4, 6 and 12 (*b*) 1, 2 and 3 (*c*) 1, 2 and 4 (*d*) None of the above

9. The number of linearly independent solutions of the homogenous system of equations AX = 0 where X consists of *n* unknowns and A consists of *m* linearly independent rows is

(*a*) $(m - n)$ (*b*) m

(*c*) $n - m$ (*d*) None of these

10. Consider the two statement regarding two matrices A and B of order '*n*'.

(*i*) $(A + B) \leq \text{Rank}(A) + \text{Rank}(B)$

(*ii*) $\text{Rank}(AB) \geq \text{Rank}(A) + \text{Rank}(B) - n$

(*a*) (*i*) is correct but (*ii*) is incorrect (*b*) (*i*) is incorrect but (*ii*) is correct

(*c*) both (*i*) and (*ii*) are correct (*d*) both (*i*) and (*ii*) are incorrect

11. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ 4 & 2 & 3 \end{bmatrix}$ and $1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Then which is the following is a null matrix

(*a*) $A^3 - 4A^2 + A + 6I$ (*b*) $A^3 + 4A^2 - A - 6I$

(*c*) $A^3 - 4A^2 - A + 6I$ (*d*) $A^3 - 4A^2 - A - 6I$

12. The matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is similar to the matrix

(*a*) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (*b*) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (*d*) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & 4 \end{bmatrix}$

13. If the constant term in the characteristic polynomial of a square matrix is other than zero. Then the matrix is

(*a*) Necessarily singular (*b*) Always non singular

(*c*) Can't not say (*d*) Data insufficient

14. The number of linearly independent entries in a skew symmetric matrix of order *n* equals.

(*a*) n (*b*) $\frac{n(n+1)}{2}$ (*c*) $\frac{n(n-1)}{2}$ (*d*) $n^2 - 1$

15. $A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then the determinant of $A^{100} - 2^{100}\ 1$ equals.

(*a*) 2^{100} (*b*) 2^{200} (*c*) 0 (*d*) none of these

16. Which is the following is not an elementary matrix

(*a*) $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ (*b*) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (*d*) $\begin{bmatrix} 1 & 1 \\ 0 & 5 \end{bmatrix}$

17. The number of possible sub matrices of a matrix or order 5 × 6 equals.

(*a*) 465 (*b*) 1953 (*c*) 2048 (*d*) 30

18. Rank of a matrix where all element is equal is

(*a*) 1 (*b*) 0 (*c*) 2 (*d*) None of these

19. If A is an *n*-rowed square matrix of rank (*n* – 1). Then

(*a*) Every co-factor of A is non zero (*b*) $\text{Adj } A \neq 0$

(*c*) A does not exist (*d*) None of these

20. Rank of a skew symmetric matrix cannot be

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 0

21. If A and B two ($n \times n$) type matrices and P and Q are two non singular matrices. Then if rank of A is r then rank of B will be

(a) r (b) $(n - r)$ (c) n (d) $(n - 2r)$

22. If two matrices have same rank. Then

(a) They are equivalent
(b) They will be equivalent if they are of same size.
(c) They cannot be equivalent
(d) None of these

23. Matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$

(a) A is equivalent to any matrix of rank 3
(b) A is equivalent to I_3
(c) A is equivalent to any matrix having rank ≤ 3
(d) A is equivalent to any matrix of rank ≥ 2

24. If A and B two matrix then

(a) Rank (AB) = (Rank A) (Rank B)
(b) Rank(AB) $\leq$ (Rank A) or Rank(AB) $\leq$ Rank(B)
(c) Rank(AB) $\leq$ Rank B
(d) Rank(AB) = Rank A = Rank B

25. If A is ($n \times 1$) column matrix and B is $1 \times n$ row matrix. Then rank (AB) equals

(a) n (b) $\leq n$ (c) 1 (d) Does not exist

26. If A is any matrix where rank is equal to r and B is any non-singular matrix then

(a) Rank (AB) = Rank (A) = Ran (B)
(b) Rank(AB) = Rank (A)
(c) Rank (AB) = Rank (B)
(d) Rank(AB) $\leq$ Rank(A)

27. A and B are two matrices of same order then

(a) Rank(A + B) = Rank A + Rank B
(b) Rank(A + B) $\leq$ Rank A + Rank B
(c) Rank(A + B) $\geq$ Rank A + Rank B
(d) Rank(A + B) = Rank A + Rank B

28. A be 3×3 matrix and Rank of A^3 is 2. Then rank of A^6 will be

(a) 2 (b) 3 (c) 1 (d) $1 \leq 2$

29. Consider the following matrix $A = \begin{bmatrix} 2 & 3 \\ X & Y \end{bmatrix}$

If the eigen values of A are 4 and 8 then

(a) X = 4, Y = 10
(b) X = 5, Y = 8
(c) X = –3, Y = 9
(d) X = –4, Y = 10

30. Consider the matrix as given below $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$

Which one of the following options provides the correct values of the eigen values of the matrix.

(a) 1, 4, 3 (b) 3, 7, 3 (c) 7, 3, 2 (d) 1, 2, 3

ANSWER KEY

1	2	3	4	5	6	7	8	9	10
c	b	a	d	c	b	c	d	c	c
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
a	a	b	c	c	d	b	b	b	a
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
a	b	b	b	c	b	b	a	d	a

SOLUTIONS

1. Write the given equation in matrix form as AX = B and we find that Rank[A : B] = 3
and Rank[A] = 2
So, $\text{Rank}[A : B] \neq \text{Rank}[A]$; the system is in consistent so it does not posses a solution

2. As for a matrix we have

$$A(\text{Adj } A) = |A|I$$

I being identity matrix and | A | being determinant of matrix A ; Adj A being Adjoint of matrix A by taking determinant of both sides

$$|A(\text{Adj } A)| = ||A|| I| = |A|\,|\text{Adj } A| = |A|^n$$

n being on order of the matrix

$$\Rightarrow \quad |\text{Adj } A| = |A|^{n-1}$$

$$|\text{Adj Adj } A| = |\text{Adj } A|^{n-1} = (|A|^{n-1})^{n-1}$$

Also $\quad |A^{-1}| = \dfrac{1}{|A|}$

So by putting order $n = 3$, $|A| = 2$

$$\Rightarrow \quad |\text{Adj Adj } A^{-1}| = \left(\left(\frac{1}{2}\right)^2\right)^2 = \frac{1}{4}$$

3. By taking conjugate A symmetric matrix remains symmetric as conjugate of each element is taken when A and B symmetric matrices. Then so (A + B) and hence so is conjugate of (A + B).

4. As A and B both are not square matrices, so their determinant does not exist.

5. If A is an orthogonal matrix

$$AA' = I$$

$$|AA'| = |I|$$

$$|A||A'| = I \qquad |A|^2 = 1$$

$$\boxed{|A| = \pm 1}$$

6. Matrix multiplication is always associative but it is not necessarily always Abelian or non Abelian

7. $AA^{\theta} = 1$

$$|A||A^{\theta}| = 1$$

But as conjugate $\quad |A| = |A^{\theta}|$

So we get $\quad ||A||^2 = 1$

$\Rightarrow$ Modulas of | A | = equal to 1.

8. As the product of eigen values of a matrix is equal to the determinant of the matrix but here we have eigen values as 1, 2, 3 whole product is 6 which is not equal to determinant of the matrix which is equal to 12. Hence this is not possible.

9. If matrix A consists of *m* linearly independent rows. Then it is Rank will be *n* and for Homogeneous system of equations AX = 0 we have number of linearly independent solutions as $(n - m)$ where *n* is the the no. of unknowns and *r* is the rank of coefficient matrix.

10. $\text{Rank}(A + B) \le \text{Rank}(A) + \text{Rank}(B)$ as by adding two matrices number of linearly independent rows of both of the may or may not be retained as linearly independent rows.
Also we have

$$\text{Rank}(AB) \ge \text{Rank}(A) + \text{Rank}(B)$$

n being the order of there two matrices

11. **Cayley Hamilton Theorem:** Every matrix satisfies. It's quadratic equation so find $|A - \lambda I| = 0$. Which is the characteristic equation. In this equation replace λ by A to get the value corresponding to the null matrix on the right.

12. Similarly matrices have same set of eigen values so identify it by this rule.

13. As product of eigen values of a matrix has it is magnitude equal to the constant term, so product is non zero. Hence each eigen value is equal to a non zero value. Hence determinant will be equal to non zero value; as determinant is equal to product of the eigen values.

14. In skew-symmetric matrix entries along principal diagonals are equal to zero and along non diagonals entries are allotted in pairs one of them being opposite in sign to the other. So for matrix of order *n*. The *n* entries along principal diagonal have no freedom rest $(n^2 - n)$ non diagonal elements have freedom in pairs. So, there will be $\frac{(n^2 - n)}{2}$ free entries = $\frac{n(n-1)}{2}$

15. Eigen values of matrix A are 2, 2. Hence eigen values of $(A^{100} - I^{100})$ will be $(2^{100} - 2^{100})$ and $(2^{100} - 2^{100})$ 0 as 0. Hence determinant which is product of eigen values will be equal to zero (0).

16. An elementary matrix a matrix obtained from Identify matrix by applying a single elementary transformation on it. Matrix option (*d*) requires two elementary transformations; while others requires only one.

17. Out of 5 rows select any 1 or 2 or 3 or 4 , 4 or 5 and simultaneously any 1, 2, 3, 4, 5, 6 columns which 1^{st} 31 * 63 = 1953.

18. If eigen values of a matrix are distinct. Then it is diagonalization.

19. A^{-1} does not exist.

20. Rank cannot be equal to 1.