# SAMPLE STUDY MATERIAL

## Mechanical Engineering

## ME

## Postal Correspondence Course

# Industrial Engineering

## GATE, IES & PSUs

# CONTENT

# CHAPTER-1

# QUEUING THEORY

**Introduction:**

→ A common situation that occurs in everyday life is that of waiting in a line.

→ Waiting lines (queues) are usually seen at metro stations, bank counters, traffic lights, bus stops etc.

→ Queuing theory can be used to a variety of operational situations where it is not possible to accurately predict the service rate of service facility and arrival rate of customers.

**Queuing System:**

The queuing system can be completely described by

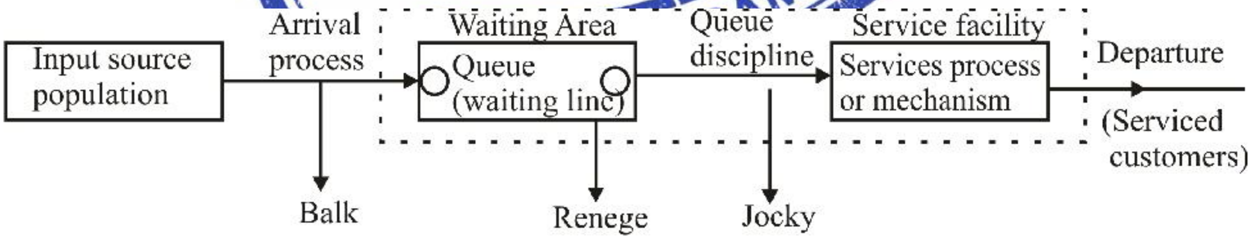
(*i*) The input (arrival pattern)

(*ii*) Queuing process

(*iii*) Queue discipline

(*iv*) Service process






→ Customers are defined as those in need of service.

→ The manner in which customers arrive at the service facility, in batches, individually, at scheduled or unscheduled time is known as the arrival pattern.

→ The customer's entry into the waiting line depends upon the queue conditions.

→ **Queue Discipline:**

Customers, from waiting line (queue), are selected for service according to certain rules known as queue discipline.

→ It is the rule according to which the customers are selected for service when a queue has been formed the most common discipline are

**(*i*)First** come first served (FCFS)

**(*ii*) First in** first out (FIFO)

**(*iii*) Last** in first out (LIFO)

**(*iv*) Selection** for service in random order (SIRO)

→ **Arrival Process:**

This element of waiting line (queuing) system is concerned with the pattern in which the customers arrive for service. It can be described by following three factors:

**(i). Size of the Queue:** If the total number of customer requiring service are only few, then size of input source is considered to be finite. Whereas, if customers requiring service are sufficiently large in number, then the input source is said to be infinite.

**(ii). Pattern of Arrivals:** In case the arrivals times are known with certainity, then queuing problems are categorized as deterministic models. Whereas, if the time between successive arrivals is uncertain, then arrival pattern is measured by mean arrival rate.

→ The most queuing models assume that arrival rate follow Poisson distribution and mean arrival pattern or inter arrival times follow an exponential distribution.

**(iii). Customer's Behavior:** A customer may decide to wait no matter how long the queue becomes, he is said to have patient customer. Other hand if the queue is too long to suit him, may decide not to enter it, he is said to have impatient customer. For impatient customers.

(*a*) If a customer decides not to enter the waiting line because of its length, he is said to have balked.

(*b*) If a customer enters the waiting line, but after some time loses his patience and decides to leave them he is said to have reneged.

(*c*) If a customer move from one waiting line to another for his personal gains, then he is said to have jockeyed for position.

**3. Service Mechanism:** It is concerned with service time and service facilities.

Service facilities are

(*a*) Single queue-one server (*b*) Several queues-one server

(*c*) Single queue-several servers (*d*) Several servers.

→ Some of the operational characteristics of a queuing system.

**1. Expected number of customers in the queue** $(L_q)$**:** It is the average number of customers waiting in the queue excluding the customer being served.

**2. Expected Number of customers in the system** $(L_s)$**:** It is the average number of customer in the system, both waiting and in service.

**3. Expected waiting time in the system(W):** It is the average total time spent by a customer in the system. It is taken to be the waiting time plus customer being served.

**4. Expected waiting time in queue $(W_q)$ :** It is the average time spent by a customer in the waiting line before the commencement of his service.

**Deterministic Queuing System:** A queuing system where in the customers arrive at regular intervals and the service time for each customer is known and constant, is called a deterministic queuing system.
**Probability Distributions in queuing system:** In queuing system customers arrive in queuing system in a random manner and follow a Poisson's distribution or equivalently the inter arrival times obey exponential distribution.

**Kendall's Notation for Representing queuing models:**
Arrival pattern/Departure pattern/No. of server/System capacity/Priority rule

M/M/1/$\infty$/FCFS

D/M/1/5/SIRO

**Classification of queuing models:** The queuing models are classified as follows:

**Model I: (M/M/1): (ã/FCFS) :**

This denotes Poisson arrival, Poisson departure, single server, infinite capacity and first come first served discipline. Letter M is used due to Markovian property of exponential process.

**Model II: Multi Service Model (M/M/S): (ã/FCFS) :**

This model takes the number of service.

**Model III: (M/M/1): (N/FCFS):** Capacity of the system (finite), say N.

**Busy Period (Traffic Intensity):** It is given by traffic intensity = $\rho = \dfrac{\text{Mean Arrival Rate}}{\text{Mean Service Rate}} = \dfrac{\lambda}{\mu}$

$\rightarrow$ **Expected Number of Units in the System:**

$$L_s = \text{(Waiting + Being served)}$$

$$L_s = \frac{\text{No. of arrival rate}}{\text{No. of service rate} - \text{No. of arrival rate}}$$

$$L_s = \frac{\lambda}{\mu - \lambda}$$

→ **Expected Number of units in the queue:**

$$L_q = \text{Expected no. of units in the system} - \frac{\text{Arrival rate}}{\text{Service rate}}$$

$$L_q = L_s - \frac{\lambda}{\mu}$$

$$\boxed{L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}} \quad \text{Average length of queue}$$

→ **Expected Waiting Time Per unit in the system:**

$$W_s = \frac{\text{Expected number of units in the system}}{\text{Arrival rate}}$$

$$W_s = \frac{1}{\mu - \lambda}$$

→ **Expected waiting time per unit in the queue:**

$$W_q = \text{Expected waiting time in system} - \text{Time in service}$$

$$W_q = W_s - \frac{1}{\mu} \qquad W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

→ **Expected Number of units in a non-empty queue:**

$$L_n = \frac{\text{Expected number in queue}}{\text{Probability that queue is not empty}} \qquad L_n = \frac{\lambda}{\mu-\lambda}$$

→ **Expected Waiting Time $W_n$ for a non-empty queue:**

$$W_n = \frac{1}{\mu-\lambda}$$

→ **Probability That *n* customer in system:**

$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n = (1-\rho)\,\rho^n$$

→ **Non Empty queue:** A queue that form time to time 24 hours operation etc or un ending queue. Average length of unending queue

# CHAPTER-2
# SEQUENCING

**Introduction:**

This chapter deals with the problem of determining the order (sequence) in which a number of jobs should be performed on various machines in order to make effective use of available facilities and achieve more output.

Consider a sequencing problem where N jobs are to be performed on K different machines. In such a case, is to determine the order (sequence), which **minimizes the total elapsed time**. Elapsed time means the time from the start of first job upto the finish of the last job.

**Discuss the following cases:**

- Processing N jobs through two machines.
- Processing N jobs through three machines.
- Processing two jobs through K machines

**General assumptions are as follows:**

- The processing time on each machine is known.
- No machine may process greater than one job simultaneously.
- Each job, once started on a machine is to be performed up to finish on that machine.
- The time required to finish a job is independent of the order of the jobs in which they are to be processed.
- The time taken by each job in changing over from one machine to another machine is negligible.
- The sequence of completion of job has no significance, i.e., no job is to be given priority.
- A job starts on a machine as soon as the job and the machine both are idle.

**Basic Terminology:**

**Processing Time:** It refers the time required by a job on each machines.

**Number of Machines:** It is refer to the number of service facilities through which a job must pass before it is assumed to be completed.

**Processing Order:** It indicates to the order in which machines are required for completing the job.

**Idle Time on a Machine:** It refers to the time during which a machine does not have a job to process.

**Total Elapsed Time:** It means that the time from the start of first job upto the completion of the last job.

**No Passing Rule:** It indicates the rule of maintaining the order in which jobs are to be processed on the given machines. *e.g.* if *k* jobs are to be processed on two machines, $M_1$ and $M_2$ in the order $M_1 M_2$, then each job should go first to machine $M_1$ and then to $M_2$.

**Job Arrival Pattern**

The pattern of arrivals into the system is (i) static (ii) dynamic.

**Static.** In this case, If certain numbers of jobs arrive simultaneously and no furthermore jobs arrive until the present set of jobs has been processed, then the problem is called static.

**Dynamic.** In dynamic case, jobs arrive after certain period of time and arrival of jobs will continue indefinitely in future also.

**Sequence of Machines:**

**Fixed sequence.** In this case, given jobs are processed in a fixed sequence. For example each job is to be processed first on machine I, then on machine II, and so on.

**Random Sequence.** In this case, given jobs are processed in a random order.

**Processing Time**

**Deterministic.** In this case, If the processing time is known with certainty, then it is a deterministic problem.

**Probabilistic.** In this case, If only expected processing time is known, it is a probabilistic problem.

**Processing n Jobs Through Two Machines:**

This problem refers to the following situation:

Consider N jobs are to be processed on two machines, say X & Y. Each job has to pass through the same sequence of operations in same order. After a job is completely processed on machine X, it is assigned to machine Y. If machine Y is not free at that moment, then the job enters the waiting queue. Each job from the waiting queue is assigned to machine Y according to FIFO discipline.

Let

$X_i$ = Processing time for $i^{th}$ job on machine X

$Y_i$ = Processing time for $i^{th}$ job on machine Y

T = Total elapsed time

The problem here is to determine the order (sequence) in which these N jobs should be processed through X & Y, so that the total elapsed time (T) is minimum.

**Example 1**

Suppose we have five jobs, each of which has to be processed on two machines X & Y in the order XY. Processing times are given in the following table:

| Job | Machine X | Machine Y |
|---|---|---|
| I | 6 | 3 |
| II | 2 | 7 |
| III | 10 | 8 |
| IV | 4 | 9 |
| V | 11 | 5 |

Calculate an order in which these jobs should be processed so as to minimize the total processing time.

**Solution.**

The minimum time in the above table is 2, which corresponds to job II on machine X.

Eliminate job II from further consideration. The reduced set of processing times are

| Job | Machine X | Machine Y |
|---|---|---|
| I | 6 | 3 |
| III | 10 | 8 |
| IV | 4 | 9 |
| V | 11 | 5 |

The minimum time is 3 for job I on machine Y. Therefore, this job should be done in last. allocation of jobs till this stage would be

| | | | | |
|---|---|---|---|---|
| *I* | | | | *II* |

After elimination of job I, the reduced sets of processing times are

| Job | Machine X | Machine Y |
|---|---|---|
| III | 10 | 8 |
| IV | 4 | 9 |
| V | 11 | 5 |

Similarly, by repeating the above steps, the **optimal sequence** is:

| | | | | |
|---|---|---|---|---|
| *II* | *IV* | *III* | *V* | *I* |

Once the optimal sequence is determined, the minimum elapsed time is calculated:

| Job | Machine X | | Machine Y | |
|---|---|---|---|---|
| | Time in | Time out | Time in | Time out |
| II | 0 | 2 | 2 | 9 |
| IV | 2 | 6 | 9 | 18 |
| III | 6 | 16 | 18 | 26 |
| V | 16 | 27 | 27 | 32 |
| I | 27 | 33 | 33 | 36 |

Idle time for machine X = total elapsed time - time when the last job is out of machine X
36 - 33 = 3 hours

Idle time for machine Y = Time at which the first job in a sequence finishes on machine X + $\sum_{1=2}^{n}$ {( time when the $i^{th}$ job starts on machine Y) - (time when the $(i-1)^{th}$ finishes on machine Y)}

Idle time for machine Y = 2 + (9 - 9) + (18 - 18) + (27 - 26) + (33 - 32) = 2+1+1=4 hours

**Processing *N* Jobs Through Three Machines:**
By applying Johnson rule a N job 3 machine case is to be converted into N job two machine case if any one of the following conditions are satisfied.
1. Minimum process time on machine A must be $\geq$ Maximum processing time on machine B.
2. Minimum processing time on C must $\geq$ Maximum processing time on machine B.
Processing time on G = Processing time on A + Processing time on C.
→ Processing time on H = Processing time on B + Processing time on C
→ If the condition is not satisfied take B as $1^{st}$ machine (A + C) as $2^{nd}$ machine, apply Johnson's rule and determine the make span time.
→ Take (B + A) as first machine and 'C' as second machine apply Johnson's rule.
Among the two times whichever is minimum that is the best schedule.

ABC Company has to process five items on three machines X, Y & Z. Processing times are given in the following table:

| Item | X | Y | Z |
|---|---|---|---|
| 1 | 4 | 4 | 6 |
| 2 | 9 | 5 | 9 |

| | | | |
|---|---|---|---|
| 3 | 8 | 3 | 11 |
| 4 | 6 | 2 | 8 |
| 5 | 3 | 6 | 7 |

Determine the sequence that minimizes the total elapsed time.

***Solution.***

Min. (X) = 3, Max. (Y) = 6 and Min. (Z) = 6. Since the condition of Max. (X) $\leq$ Min. (Z) is satisfied, The processing times for the new problem are

| Item | G = X + Y | H = Y + Z |
|---|---|---|
| 1 | 8 | 10 |
| 2 | 14 | 14 |
| 3 | 11 | 14 |
| 4 | 8 | 10 |
| 5 | 9 | 13 |

The optimal sequence is

| | | | | |
|---|---|---|---|---|
| 1 | 4 | 5 | 3 | 2 |

| Item | Machine X | | Machine Y | | Machine Z | |
|---|---|---|---|---|---|---|
| | Time in | Time out | Time in | Time out | Time in | Time out |
| 1 | 0 | 4 | 4 | 8 | 8 | 14 |
| 4 | 4 | 10 | 10 | 12 | 14 | 22 |
| 5 | 10 | 13 | 13 | 19 | 22 | 29 |
| 3 | 13 | 21 | 21 | 24 | 29 | 40 |
| 2 | 21 | 30 | 30 | 35 | 40 | 49 |

Total elapsed time = 49

Idle time for machine X = 49 – 30

= 19 hours

Idle time for machine Y= 4 + (10 - 8) + (13 - 12) + (21 - 19)+ (30 - 24) + (49 - 35)

= 4+2+1+2+6+14

=29 hours

Idle time for machine Z = 8 + (14 - 14) + (22 - 22) + (29 - 29)+ (40 - 40) = 8 hours

***Example 2***

Sambodhi Import House has to process five items through three stages of production, viz, **Facing**, **Turning** & **Drilling** . Processing times are given in the following table:

| Item | Facing X | Turning Y | Drilling Z |
|---|---|---|---|

| | | | |
|---|---|---|---|
| 1 | 3 | 3 | 5 |
| 2 | 8 | 4 | 8 |
| 3 | 7 | 2 | 10 |
| 4 | 5 | 1 | 7 |
| 5 | 2 | 5 | 6 |

Determine sequence in which these items should be processed so as to minimize the total processing time.

***Solution.***

The processing times for the new problem are

| Item | G = X + Y | H = Y + Z |
|---|---|---|
| 1 | 6 | 8 |
| 2 | 12 | 12 |
| 3 | 9 | 12 |
| 4 | 6 | 8 |
| 5 | 7 | 11 |

Thus, the optimal sequence is formed in any of the two ways.

| 4 | 1 | 5 | 3 | 2 |
|---|---|---|---|---|

| 1 | 4 | 5 | 3 | 2 |
|---|---|---|---|---|

| Item | Facing | | Turning | | Drilling | |
|---|---|---|---|---|---|---|
| | Time in | Time out | Time in | Time out | Time in | Time out |
| 1 | 0 | 3 | 3 | 6 | 6 | 11 |
| 4 | 3 | 8 | 8 | 9 | 11 | 18 |
| 5 | 8 | 10 | 10 | 15 | 18 | 24 |
| 3 | 10 | 17 | 17 | 19 | 24 | 34 |
| 2 | 17 | 25 | 25 | 29 | 34 | 42 |

Total elapsed time = 42

Idle time for **Facing** process = 42 – 25 = 17 hours

Idle time for **Turning** process = 3 + (8 - 6) + (10 - 9) + (17 - 15)+ (25 - 19) + (42 - 29)

=3+2+1+2+6+13= 27 hours.

Idle time for **Drilling** process = 6 + (11 - 11) + (18 - 18) + (24 - 24)+ (34 - 34) = 6 hours.

**Processing Two Jobs through K Machines:**

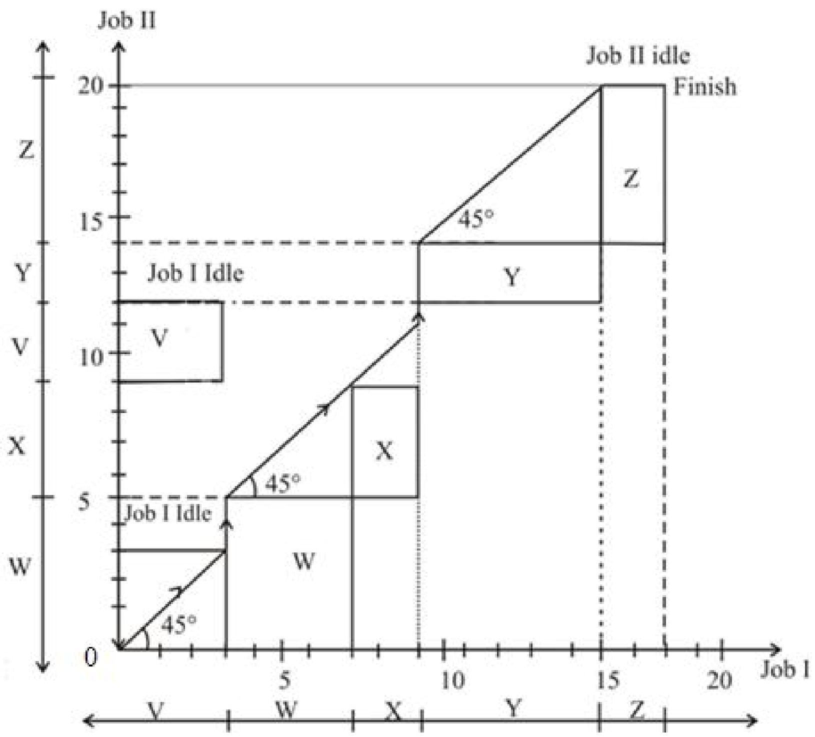
This case focuses on the problem of processing two jobs through K machines. Problems in this case, can be solved with the help of graphical method.

***Example***

Two jobs are to be performed on five machines V, W, X, Y, and Z. Processing times are given in the following table.

| | Machine | | | | |
|---|---|---|---|---|---|
| **Job I** | **V** | **W** | **X** | **Y** | **Z** |
| | **3** | **4** | **2** | **6** | **2** |
| **Job II** | **W** | **X** | **V** | **Y** | **Z** |
| | **5** | **4** | **3** | **2** | **6** |

Use graphical method to determine the total minimum elapsed time.




- Start from point 0, move through the $45^0$ line until a point marked finish is obtained.
- The elapsed time can be determined by adding the idle time for either job to the processing time for that job.
- idle time for job I is 5 hours.

  Elapsed time = Processing time of job I + Idle time of job II

  = (3 + 4 + 2 + 6 + 2) + 5 = 17 + 5 = 22 hours.
- Similarly, idle time for job II is 2 hours.

  Elapsed time = Processing time of job II + Idle time of job II

  = (5 + 4 + 3 + 2 + 6) + (2) = 20 + 2 = 22 hours.
-