SAMPLE STUDY MATERIAL

Instrumentation Engineering

IN

Postal Correspondence Course

GATE & PSUs

Network Theory

CONTENT

CHAPTER-1

BASIC CIRCUIT ELEMENTS & THEORY

1. INTRODUCTION TO CIRCUIT ELEMENT:

The circuit elements can be divided into two parts:

1. **Active Elements:** When the element is capable of delivering the energy, it is called active element.

Example: Voltage source, Current source, Transistor, Diode, Op-amp etc

2. **Passive Elements:** When the element is not capable of delivering the energy, it is called passive element.

Example: Resistance, capacitor, inductor etc.

Resistance: The ohm's law can be given as:

The voltage across a two terminal of a network is directly proportional to the current flowing through it as:

$$V \propto I$$

or, $\boxed{V = IR}$

This constant of proportionality is called 'resistance'.

Key Points:

- Power in resistor is given by

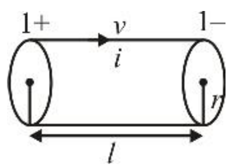
$$\boxed{P = vi = i^2R = \frac{v^2}{R}}$$

- Energy is then determined as the integral of instantaneous power as :

$$\boxed{E = \int_{t_1}^{t_2} Pdt = R\int_{t_1}^{t_2} i^2dt = \frac{1}{R}\int_{t_1}^{t_2} v^2dt}$$

- Resistance consumes energy and converts electrical energy into heat energy.
- Resistance depends on the geometry of material and also on nature of material as:

$$\boxed{R = ° \frac{l}{A}}$$



Where ° = Resistivity ($\Omega.m$)

° = 1/σ (σ = conductivity)

Unit of conductivity: mho/m or siemens/cm

- If length of wire is doubled and Radius is halved, then resistance of wire becomes 8 times larger.
- Resistivity of wire is materialistic property i.e. It does not vary with circuit geometry.
- Extension of wire result in increase in length & decrease in cross-sectional area therefore resistance of wire increases.
- When circuit is short circuit means, $R = 0$. When circuit is open, $R = \infty$.

Example: A 4Ω resistor has a current $i = 2.5\sin(\mu t)A$. Find the voltage, Power and energy over one cycle. $\mu = 500$ rad/sec

Solution: Given that $i = 2.5\sin(\mu t)A$

$V = iR$

$V = 2.5\sin(\mu t) \times 4 = 10\sin(\mu t)$ volt

$P = i^2R = [2.5\sin(\mu t)]^2 \times 4$

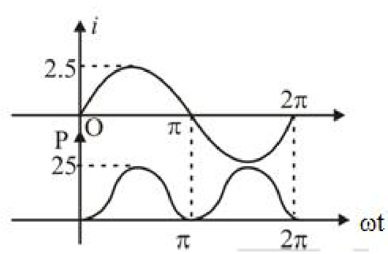
$\boxed{P = 25\sin^2(\mu t) \;\; W}$

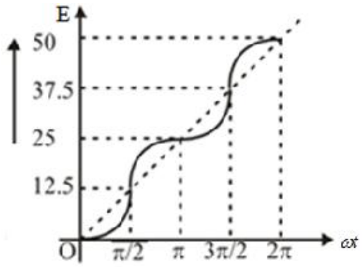
Energy $E = \int_{t_1}^{t_2} Pdt = \int_{t_1}^{t_2} 25\sin^2 \mu t\, dt$

$$E = 25\int_{t_1}^{t_2} \frac{1}{2}[1 - \cos 2\mu t]dt$$

$$\boxed{E = 25\left[\frac{t}{2} - \frac{\sin 2\mu t}{4\mu}\right] J}$$

The plot of i, P and E is as shown:





This illustrate that P is always positive and that the energy is always increasing. This is the energy dissipated by resistor.

2. CAPACITANCE:

The circuit element that stores energy in an electric field is called capacitor. When variable voltage is applied to a terminal of capacitor, the energy is stored during one part of cycle and discharge during next half cycle.

The charge across the capacitor is directly proportional to the applied voltage:

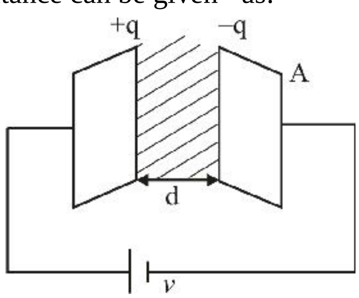
$$Q \propto V$$

$$Q = CV$$

$$\boxed{C = \frac{Q}{V}} \qquad \boxed{\text{Unit of } C \Rightarrow Farad}$$

Key Points:

(a.) **<u>Capacitors retain the charge & thus electric field</u>** after removal of the source applied. (While inductors do not retain energy). For parallel plate capacitor, the capacitance can be given as:



$$\boxed{C = \frac{\epsilon_o \epsilon_r A}{d}}$$

Where A = cross-sectional area of plate

$\epsilon_r =$ Relative permittivity of dielectric

$\epsilon_o =$ Permittivity of free space

d = distance between plates

$$\boxed{C = \frac{8.854 \epsilon_r A}{d} pF}$$

(b.) The charge q on capacitor results in an electric field in the dielectric which is the mechanism of energy storage.

(c.) Power and energy relation for capacitance are as:

$$P = vi = vc\frac{dv}{dt} = \frac{d}{dt}\left[\frac{1}{2}cv^2\right] \qquad \left\{i = \frac{cdv}{dt}\right\}$$

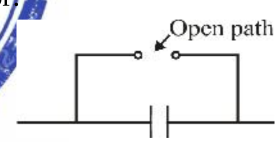
$$\boxed{P = \frac{d}{dt}\left[\frac{1}{2}cv^2\right]}$$

Energy $w_c = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} v\,c\frac{dv}{dt}dt$

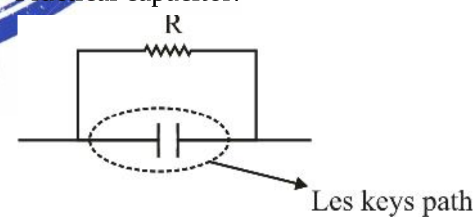
$$\boxed{w_c = \frac{1}{2}c\left[v_2^{\,2} - v_1^{\,2}\right]}$$

(d.) The energy stored in the electric field of capacitance is $\boxed{w_c = \frac{1}{2}cv^2}$

(e.) Ideal capacitor:



Practical capacitor:



Example: In the interval 0 > t > 5π ms, a 20 μf capacitor has a voltage $V = 50\sin 2(\mu t)V$. Obtain the charge, power and energy. Plot work w_c assuming $w = 0$ at $t = 0$. $\mu = 100 rad./\sec$

Solution: $q = cv = 20 \times 50 \sin 2\mu t$

$$\boxed{q = 1000 \sin 2\mu t \;\mu C}$$

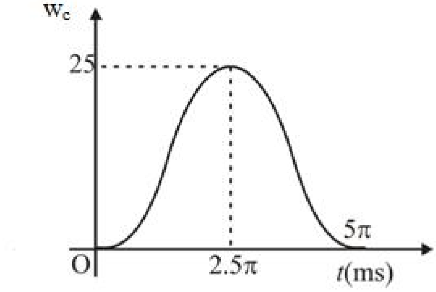
$$i = c\frac{dv}{dt} = 0.2\cos 2\mu t \;\; A$$

$$P = vi = 50\sin 200t \times 0.2\cos 2\mu t$$

$$P = 5\sin 4\mu t \;\; W$$

$$w_c = \int_{t_1}^{t_2} P\,dt = \int_0^t 5\sin 400t\,dt$$

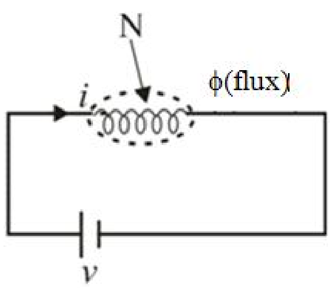
$$\boxed{w_c = 12.5\left[1-\cos 400t\right] mJ}$$



This indicates that in the internal 0 > t > 2.5π ms, the energy is stored to value of 25 mJ and then it returns to zero, as the energy is returned to the source.

3. INDUCTANCE:

The circuit element that stores energy in a magnetic field is called an inductor.



When voltage is applied across a inductor, the flux is induced in the conductor which is proportional to current flowing through it, i.e.

$$N ¤ \propto i$$

$$N ¤ = L i$$

$$\boxed{L = \frac{N¤}{i}}$$

$L \rightarrow$ Inductance

Unit$\rightarrow$ Henry

Key Points:

(a) The flux linkage across inductor is $N\phi$. Thus

$$\boxed{N¤ = LI}$$

(b) Inductor do not store energy when the connected source is removed

Voltage induced across the inductor is

$$\boxed{V = L\frac{di}{dt}} \quad \ldots\ldots\ldots\ldots\text{(A)}$$

{Voltage is induced when flux is varying}

(c) For an inductor of N no of turns:

$$\boxed{L = \frac{M_0 N^2 S}{l}}$$

S = Cross sectional area

l = Length of coil

(d) **Proof of equation A:** According to faraday's law, the emf induced across a inductor is directly proportional to the rate of change of flux through it.

$$e = -N\frac{d¤}{dt} \quad \text{\{N = no of turns in the coil\}}$$

$$e = -N\frac{d}{dt}\left\{\frac{LI}{N}\right\}$$

$$\boxed{e = -L\frac{dI}{dt}}$$

-ve sign indicates the opposition caused by emf to change of flux (Lenz's Law)

(e) The power across the inductor is:

$$\boxed{P = vi = L\frac{di}{dt}i = \frac{d}{dt}\left[\frac{1}{2}Li^2\right]}$$

(f) Energy: $w = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} Li\,dt$

$$\boxed{w = \frac{1}{2}L\left[i_2^2 - i_1^2\right]}$$

Energy stored in magnetic field by inductor is $\boxed{w = \frac{1}{2}Li^2}$

Example:

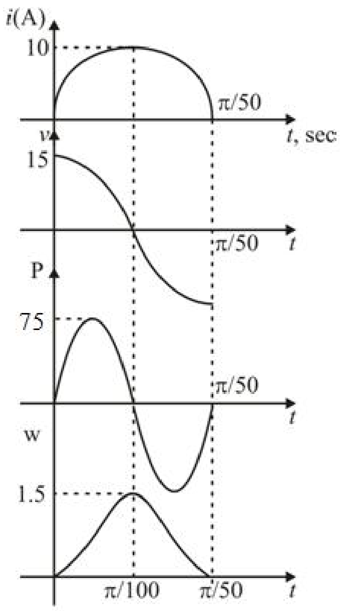
In the interval $0 > t > \left(\frac{®}{50}\right)s$, a $30\,mH$ inductance has a current i=10sin50tA. Obtain the Voltage, Power & Energy for the inductance.

Solution: $v = L\frac{di}{dt} = 15\cos 50t\ V$

$$p = vi = 75\sin(100t)W$$

$$w = \int_0^t P\,dt = \int_0^t 75\sin 100t\ dt$$

$$\boxed{w = 0.75(1-\cos 100t)\ J}$$



The Plot indicate that the energy is zero at $t = 0$ and $t = \frac{®}{50}$ sec. Thus while energy transfer occurred over the interval, this energy was first stored and later returned to the source.

Relationship of parameters:

Element	Units	Voltage	Current	Power
$+\ R\ -$, i Resistance	Ohms (Ω)	$v = Ri$ (ohms law)	$i = \frac{v}{R}$	$P = vi$ $= i^2R$
$+\ v\ -$, i inductance	Henry (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt$	$P = vi$ $= i\frac{Ldi}{dt}$
$+\ v\ -$, i Capacitance	Farad (F)	$v = \frac{1}{c}\int i\,dt$	$i = c\frac{dv}{dt}$	$P = vi$ $= vc\frac{dv}{dt}$

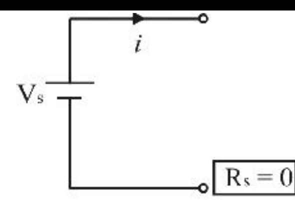
Voltage & Current Source:

The sources are of two types, one is independent sources and other is dependent sources:

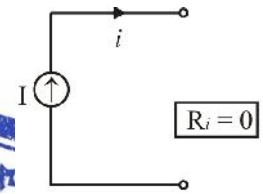
Independent sources:

The voltage or current source in which the value of voltage or current remains constant, and does not vary with other circuit element.

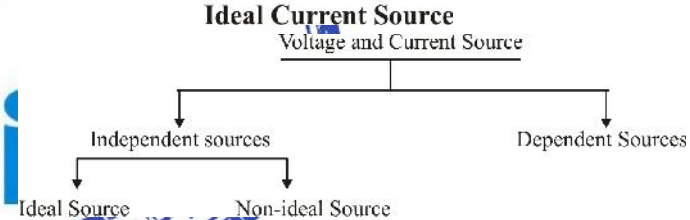
Ideal voltage and current sources:



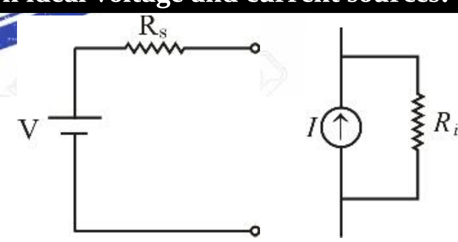
Ideal Voltage Source



Ideal Current Source



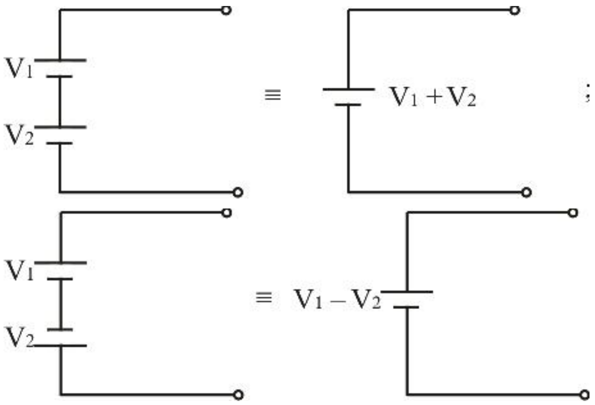
Non ideal voltage and current sources:



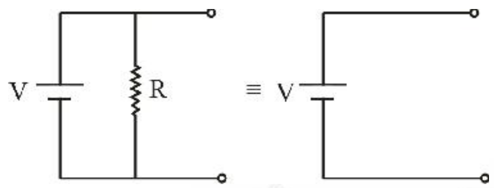
(a) In non ideal voltage source, the internal resistance of voltage source is of finite value and is always in series with voltage source.

(b) In non ideal current source, the internal resistance of current source is of finite value & is always in parallel with current source.

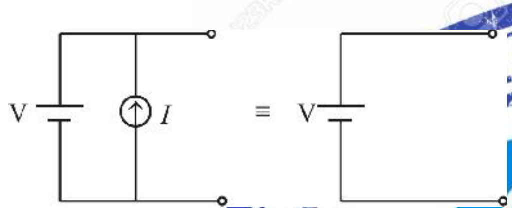
(c) V-source in series



(d)

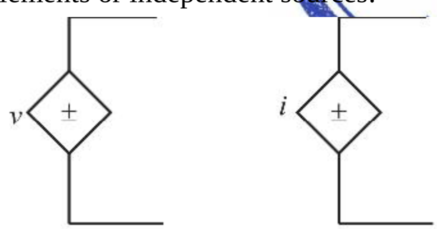


(e)

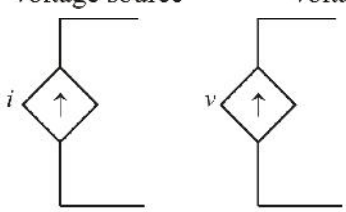


Dependent Voltage and Current Sources:

These are voltage and current sources whose value do not remain constant, rather varies with circuit elements or independent sources:



Voltage dependent voltage source   Current dependent voltage source



Current dependent current source   Voltage dependent current source

Distributed and Lumped Network:

In Lumped network, we can separate resistance, inductance, and capacitance separately or single element in one location is used to represent a distributed resistance.

Example: A coil having large number of turns of insulated wire has resistance throughout the length of wire but only resistance at single plane represents the distributed resistance.

In Distributed network, the circuit elements are not at one location rather they are distributed.

Example: Transmission line, the resistance, inductance and capacitance are distributed throughout the length of Transmission line.

Note: In distributed network, the circuit elements are represented as per unit length.

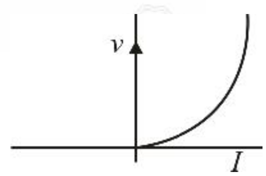
Non Linearity of circuit elements:

1. **Resistance Non Linearity:** If the current voltage relationship in an element is not linear, then the element is modeled as non linear resistor.

   Example: Diode, filament lamp (This at higher voltage droves proportional less current) etc.

   (a) The non linear resistance can be given as:

   $$R = \frac{\Delta V}{\Delta I}$$

   Note: Ohm's law is valid for linear circuit elements. Also it is not valid for open circuit element because for open circuit:

   $I = 0, R = \infty$

   So $V = \infty$

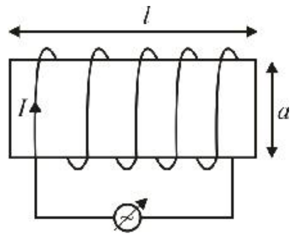
   $$V \neq IR$$

2. **Inductors non linearity:** When the inductance of inductor depends on the current magnitude, then the inductor is called non linear inductor:

   Example: Iron core inductor.

   Only air core inductor linear

Key points:



(a) $\boxed{N¤ = LI \Rightarrow L = \frac{N¤}{I} = Variable}$

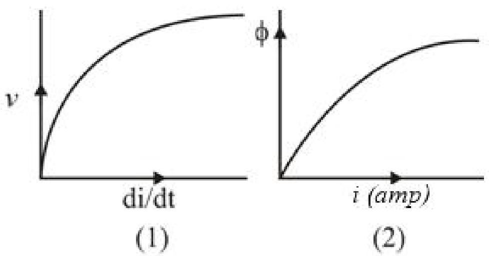
$\left\{ L = -\frac{«a}{l} \right\}$

$¤ = \frac{MMF}{S} \quad \left\{ S = \text{Reluctance} = \frac{l}{a«} \right\}$

$¤ = \frac{NIa«}{l} \Rightarrow \boxed{L = \frac{N^2 «a}{l}}$ Self inductance

(b) Also we know;

$\boxed{V = L\frac{di}{dt} \Rightarrow L = \frac{V}{di/dt} = Variable}$



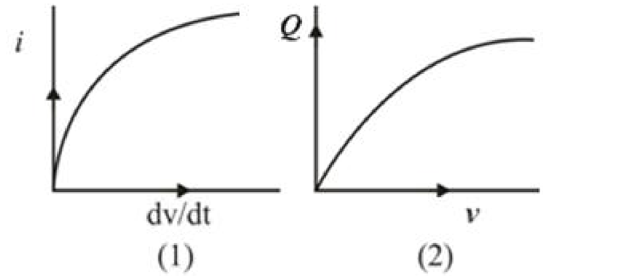
As the slope of the curve in both cases is **L** (inductance) and **L** is variable. So, the curve is not linear.

The second curve shows that after certain value of current, the flux does not increase due to saturation of iron core.

3. **Nonlinearity in capacitance:** When the capacitance of capacitor depends on voltage magnitude, then capacitor is called non linear capacitor.

$\boxed{Q = CV \Rightarrow C = \frac{Q}{V} = Variable}$

$\boxed{i = C\frac{dv}{dt} \Rightarrow C = \frac{i}{dv/dt} = Variable}$



As the slope of the curve in both cases is **C** (inductance) and **C** is variable. So, the curve is not linear.

Average and rms (effective value):

Average value: The general periodic function $y(t)$ with period T has an average value Y_{av} given as:

$$\boxed{Y_{av} = \frac{1}{T}\int_O^T y(t)dt}$$

Rms Value: The general periodic function $y(t)$ with period T has rms value Y_{rms} given as:

$$\boxed{Y_{rms} = \sqrt{\frac{1}{T}\int_O^T y^2(t)dt}}$$

Example: Find the average and rms value of following sine series

$$y(t) = a_0 + b_1 \sin µt + b_2 \sin µt + ...$$

Solution:

$$Y_{av} = \frac{1}{T}\left\{\int_0^T [a_0 + b_1 \sin µt + b_2 \sin µt + ...]dt\right\}$$

$$Y_{av} = \frac{1}{T}\{a_0 T + 0 + 0......\}$$

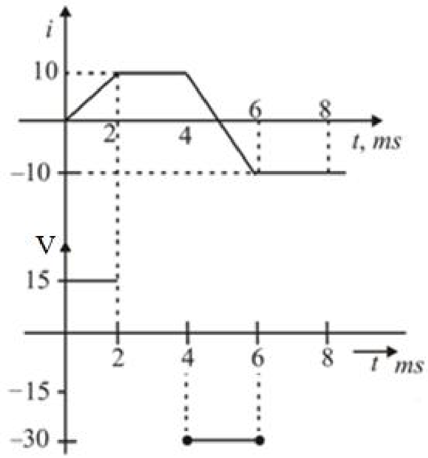
$$\boxed{Y_{av} = a_0}$$

$$Y_{rms} = \sqrt{\frac{1}{T}\int_0^T [a_0 + b_1 \sin µt + b_2 \sin µt + ...]^2 dt}$$

$$Y_{rms} = \sqrt{\frac{1}{T}\int_0^T \left[a_0{}^2 + \frac{1}{2}\left(b_1^2 + b_2^2 + ...\right) + ...\right]dt}$$

$$\boxed{Y_{rms} = \sqrt{a_0^2 + \frac{1}{2}b_1^2 + \frac{1}{2}b_2^2 + ...}}$$

Example: A single circuit element has the current and voltage functions graphed in figure below. Determine the element.



Solution: The element cannot be a resistor since V and I are not proportional. In a capacitor V is integral of i, but in interval 2ms < t < 4msec, V is not integral of i, hence element cannot be capacitor.

For inductor $V = \frac{Ldi}{dt}$

For interval 0 < t < 2ms:

$V = 15V$ and $\frac{di}{dt} = 10/2 = 5$

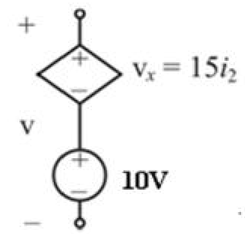
Thus $L = \frac{V}{di/dt} = 3mH$.

Thus the element is inductor. (Examine the interval 4 < t < 6 ms L must be same)

Example: Obtain the voltage V in the branch shown in figure for

(a) $i_2 = 1A$ (b) $i_2 = -2A$ (c) $i_2 = 0A$.

Solution: $v = 10 + v_x$ for



(a) $i_2 = 1A$

$V = 10 + 15 = 25V$

(b) $V = 10 - 15 \times 2 = -20V$.

(c) $V = 10 + 15 \times 0 = 10V$.

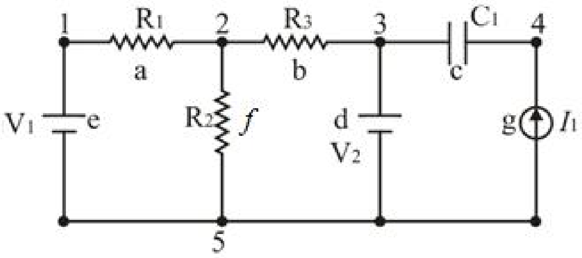
CHAPTER-2

NETWORK LAWS AND THEOREMS

1. SOME BASIC TERMS:

1. **Node:** Any point in a circuit where the terminals of two or more elements are connected together.
2. **Branch:** A branch is a part of circuit which extends from one node to other. A branch may contain one element or several elements in series. It has two terminals.
3. **Essential Node:** If three or more elements are connected together at a node, then that node sometimes called essential node.
4. **Mesh:** Any closed path which contains no other path within, called mesh.
5. **Loop:** A path which contain more than two meshes, called a loop. Thus a loop contains meshes but a mesh does not contain loop.

Example 1: Consider the following circuit:



(a) Point 1, 2, 3, 4, and 5 are nodes.

(b) a, b, c, d, e, f and g are branches.

(c) Meshes are: 1 2 5, 2 3 5, 3 4 5

(d) Loop are: 1 2 3 5 1, 2 3 4 5 2, 123451

(e) Essential node: 2, 3, 5

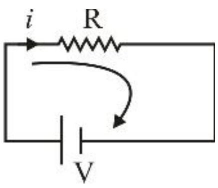
2. KIRCHHOFF'S VOLTAGE LAW:

For any closed path in a network, Kirchhoff Voltage Law (KVL) state that the algebraic sum of the voltage is zero.

Key points:

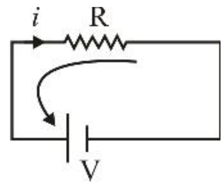
(a) $\sum v(t) = 0; \textit{for}\ \text{Closed Path}$

(b) While going in direction of current, voltage drop is taken as –ve.



$$V - iR = 0$$

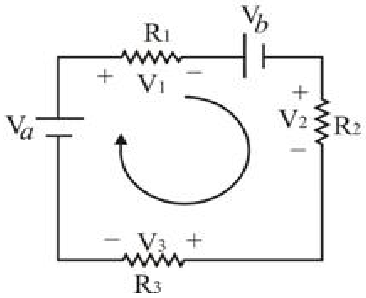
(c) While going opposite to direction of current, voltage drop across resistor is taken as positive (+ve).



$$iR - V = 0$$

(d) This law applies equally to *DC*, time variable sources.

Example: Write KVL equation for the circuit shown:



$+V_a - V_1 - V_b - V_2 - V_3 = 0$

Or $V_a - iR_1 - V_b - iR_2 - iR_3 = 0$

$V_a - V_b = i(R_1 + R_2 + R_3)$

3. KIRCHOFF'S CURRENT LAW:

KCL states that the algebraic sum of currents at a node is zero. Alternatively the sum of currents entering a node is equal to sum of currents leaving that node.

Key Points:

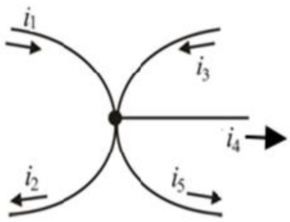
(a) It is based on the conservation of electric charge.

(b) $\sum i(t) = 0$

(c) Sign convention is arbitrary.

(d) Current entering node $\rightarrow$ are assigned +ve sign and current leaving node $\rightarrow$ are assigned –ve sign.

Example: Write the KCL equation for the principal node shown in fig below:



Solution: Principal node: Same as essential node.

$i_1 - i_2 + i_3 - i_4 - i_5 = 0$

$i_1 + i_3 = i_2 + i_4 + i_5.$

4. CIRCUIT ELEMENTS IN SERIES:

The 3 passive circuit elements in series connection have same current *i*. The voltages across elements are v_1 , v_2 , v_3.

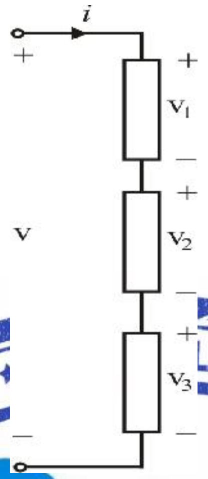
Total voltage $v = v_1 + v_2 + v_3$.

(a) **Equivalent Resistance:** When element is resistance :

$$v = i(R_1 + R_2 + R_3)$$

$$v = i\,\mathrm{R}_{eq}.$$

$$\boxed{\mathrm{R}_{eq.} = R_1 + R_2 + R_3}$$



(b) **Equivalent Inductance:** When element in above circuit is inductor then :

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt}$$

$$v = (L_1 + L_2 + L_3)\frac{di}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + L_3}$$

For any number of inductance: $L_{eq} = L_1 + L_2 + L_3 + L_4 + ...$

(c) **Equivalent Capacitance :**When the circuit element is capacitor in above circuit then,

$$v = \frac{1}{c_1}\int i\,dt + \frac{1}{c_2}\int i\,dt + \frac{1}{c_3}\int idt$$

$$v = \left(\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}\right)\int idt$$

$$v = \frac{1}{C_{eq}}\int i\,dt$$

Then $\boxed{\frac{1}{C_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}}$

5. CIRCUIT ELEMENTS IN PARALLEL:

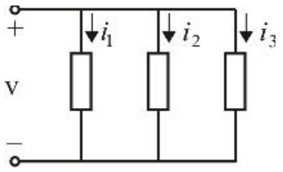
The 3 element are connected as shown in figure

(a) **Equivalent Resistance:** $i = i_1 + i_2 + i_3$

$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$$

$$i = v\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

Then $\boxed{\frac{1}{\mathrm{R}_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$



(b) **Equivalent Inductance :**

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + ...}$$

For two inductance $\boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$

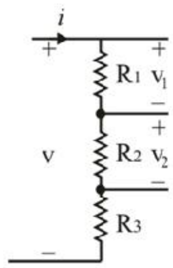
(c) **Equivalent Capacitance :**

$$\boxed{C_{eq} = c_1 + c_2 + ...}$$

This is of the same form as resistor in series.

6. VOLTAGE DIVISION:

A set of series-connected resistor is referred as a voltage divider.



This concept is applicable to *n* number of resistance.

$$v_1 = v\frac{R_1}{R_2 + R_3 + R_1}$$

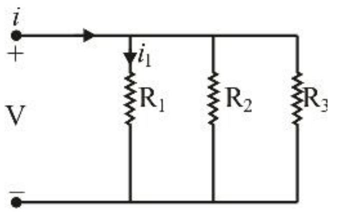
$$v_2 = v\frac{R_2}{R_1 + R_2 + R_3}$$

In voltage divider, voltage across one branch

$$=\text{Total voltage} \times \frac{\text{Resistance of that branch}}{\text{total resistance}}.$$

7. CURRENT DIVISION:

A Parallel arrangement of resistors results in a current divider.



$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} \text{ and } i_1 = \frac{v}{R_1}$$

$$\frac{i_1}{i} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Thus $$i_1 = \frac{R_2 R_3 i}{(R_1 R_2 + R_2 R_3 + R_3 R_1)}$$

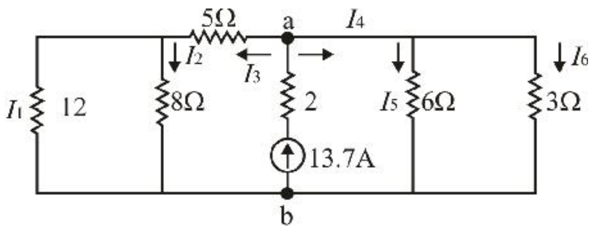
For two branch current divider:

$$i_1 = \frac{R_2 i}{R_1 + R_2}$$

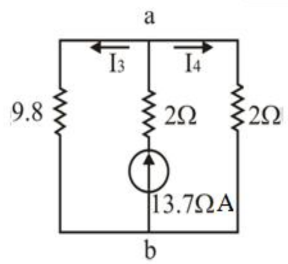
- For 2 branch circuit, the current in one branch is equal to the :

$$\text{Total current} \times \frac{\text{Resistance of Other Branch}}{\text{Total Resistance}}$$

Example: 1: Find all branch currents in the network shown below:



Solution: Circuit can be simplified as



$R_{eq.}$ (Left) $= 5 + \frac{12 \times 8}{20} = 9.8\Omega$

$R_{eq.}$ (Right) $= \frac{6 \times 3}{9} = 2\Omega$

Using current divider theorem:

$I_3 = \frac{2}{9.8 + 2} \times 13.7 = 2.32A$

$I_4 = \frac{9.8 \times 13.7}{9.8 + 2} \times 11.38A.$

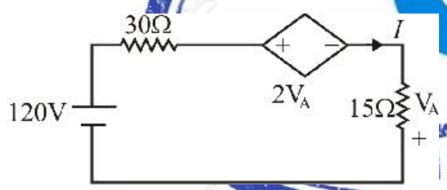
$I_3 = I_1 + I_2$

$I_1 = \frac{8 \times 2.32}{12 + 8} = 0.93A,\ I_2 = 2.32 - 0.93 = 1.39A.$

$I_4 = I_5 + I_6$

$I_5 = \frac{3 \times 11.38}{3 + 6} = 3.79A,\ I_6 = 11.38 - 3.79 = 7.59A$

Example: 2: In the circuit shown, what is the power absorbed by each element.



Solution: Writing KVL for the loop:

$120 - 30I - 2V_A - 15I = 0;\ V_A = -15I$

Thus $I = \frac{120}{15} = 8A.$

$P_{120V} = -8 \times 120 = -960W$ (Power delivered)

$P_{30\Omega} = i^2 R = 8^2 \times 30 = 1920W$ (Power absorbed)

$P_{15\Omega} = 8^2 \times 15 = 960W$ (Power absorbed)

$P_{voltage\ source} = 2V_A I = 2(-15 \times 8) \times 8 = -1920W$ (*Power delievered*)

Total power absorbed

$= -960 + 1920 + 960 - 1920 = 0W.$

Note: Power absorbed by source = – (Power delivered by source)

77 Final Selections in Engineering Services 2014.

Rank	Roll	Name	Branch
1	171298	SAHIL GARG	EE
3	131400	FIRDAUS KHAN	ECE
6	088542	SUNEET KUMAR TOMAR	ECE
8	024248	DEEPANSHU SINGH	EE
10	207735	VASU HANDA	ECE
22	005386	RAN SINGH GODARA	ECE
22	032483	PAWAN KUMAR	EE
29	070313	SAURABH GOYAL	EE
31	214577	PRAMOD RAWANI	EE
33	075338	DIPTI RANJAN TRIPATHY	ECE
35	003853	SHANKAR GANESH K	ECE
35	091781	KOUSHIK PAN	EE
36	052187	ANOOP A	ECE
37	008233	ARPIT SHUKLA	ECE
38	106114	MANISH GUPTA	EE
41	018349	VINAY GUPTA	ECE
44	098058	LEENA P MARKOSE	EE
45	029174	NAVNEET KUMAR KANWAT	EE

9 Rank under AIR 100 in GATE 2015 (Rank 6,8,19,28,41,56,76,91,98)

and many more............................

To Buy Postal Correspondence Package call at 0-9990657855