SAMPLE STUDY MATERIAL

Electronics Engineering

EC / E & T

Postal Correspondence Course

GATE, IES & PSUs

Electromagnetic Theory

ENGINEERS INSTITUTE OF INDIA
India's Best Institute for IES, GATE & PSUs
A Team of IES & GATE Toppers

CONTENT

CHAPTER-1

COORDINATE SYSTEM

The physical quantity we dealing in electromagnetic are function of space and time, to describe the spatial variations of these quantities we have to define all points uniquely in space in a suitable manner, and this require an appropriate co – ordinate system.

We deal with ***orthogonal system***. The orthogonal system is one in which the co–ordinates are mutually perpendicular.

Example: Cartesian (or Rectangular), circular cylindrical, spherical.

1. CARTESIAN CO–ORDINATE SYSTEM OR RECTANGULAR COORDINATE SYSTEM

A point P in Cartesian co-ordinate system is represented as, P (X, Y, Z) and ranges of the co–ordinate variable X, Y and Z are:

$-\infty < X < \infty$

$-\infty < Y < \infty$

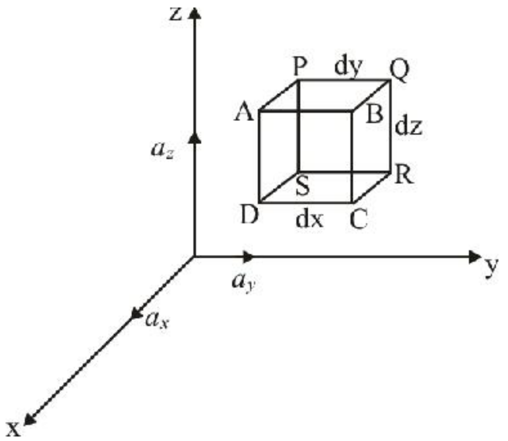
$-\infty < Z < \infty$

A vector $\vec{A}$ in Cartesian co–ordinate system, can be written as, unit vector is also re

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

Where, $\hat{a}_x, \hat{a}_y, \hat{a}_z$ unit vector along the X, Y and Z directions.

Differential Length, Area, and Volume:



1. Differential length : dl

$$dl = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

2. Differential Area: ds

$$ds = dydz\hat{a}_x = dxdz\hat{a}_y = dxdy\hat{a}_z$$

ds is a vector quantity.

Example : If we move P to Q , $dl = dy\hat{a}_y$

If we move D to Q , $dl = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$

3. Differential volume: dv

$$dv = dxdydz$$

dv is a vector quantity.

For surface ABCD. $ds = dydz\hat{a}_x$

For surface PQRS. $ds = dydz(-\hat{a}_x)$

2. CIRCULAR CYLINDRICAL CO–ORDINATES

This co–ordinate system is used when we have problem having cylindrical symmetry.

A point P in this co–ordinate system is represented as, P (° ,¤ , Z), and range of values of co–ordinate variable ° , ¤ , Z are

O ≤ ° < ∞ ; ° is the radius of cylinder passing through P.

O ≤ ¤ < 2® ; ¤ is called the azimuthal angle is measured from the X-axis in the XY plane

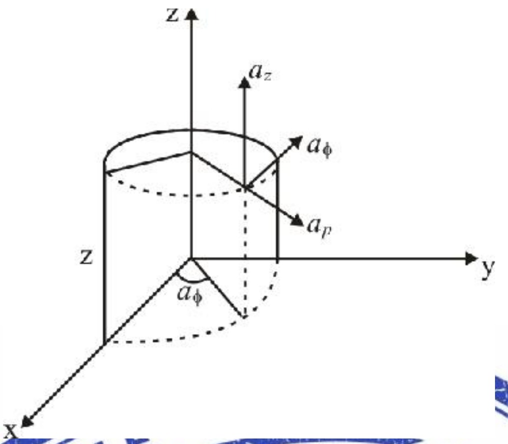
–∞ < Z < ∞ ; Z is same as in Cartesian system.

A vector $\vec{A}$ in cylindrical co – ordinates is written as,

$$\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

Where $\hat{a}_\rho$, $\hat{a}_\phi$ *and* $\hat{a}_z$ are unit vector in the ρ, ϕ, and Z direction.

Note : $\hat{a}_\phi$ is not in degrees, it assumes the units of A.



$\hat{a}_\rho$, $\hat{a}_\phi$, $\hat{a}_z$ are mutually perpendicular, and

a_ρ Points in the direction of increasing ρ.

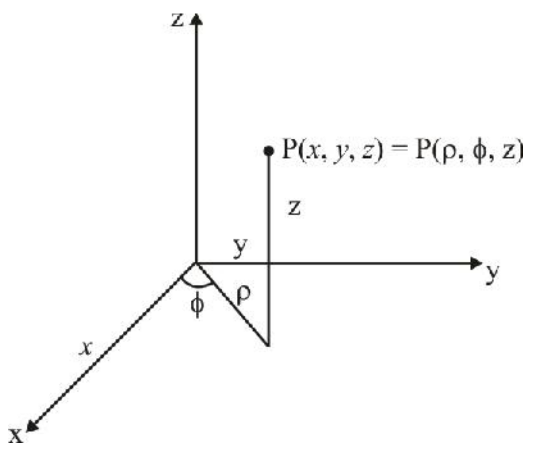
a_ϕ Points in the direction of increasing ϕ.

a_z Points in the direction of positive Z direction.

$$\hat{a}_\rho \cdot \hat{a}_\rho = \hat{a}_\phi \cdot \hat{a}_\phi = \hat{a}_z \cdot \hat{a}_z = 1$$ $$\hat{a}_\rho \cdot \hat{a}_\phi = \hat{a}_\phi \cdot \hat{a}_z = \hat{a}_z \cdot \hat{a}_\rho = 0$$

$$\hat{a}_\rho \cdot \hat{a}_\phi = \hat{a}_z$$ $$\hat{a}_\phi \cdot \hat{a}_z = \hat{a}_\rho$$ $$\hat{a}_z \cdot \hat{a}_\rho = \hat{a}_\phi$$

Relation between Cartesian co–ordinate System Variable and Cylindrical Co–ordinate System Variable



$$\cos\phi = \frac{x}{\rho} \quad ; \quad \sin\phi = \frac{y}{\rho}$$

$$x = \rho\cos\phi \quad ; \quad y = \rho\sin\phi$$

$$\therefore \boxed{x^2 + y^2 = \rho^2} \qquad \therefore \boxed{\rho = \sqrt{x^2+y^2}}$$

$$\tan\phi = \frac{y}{x} \quad ; \quad \boxed{\phi = \tan^{-1}\frac{y}{x}}$$

$$\hat{a}_x = \cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi \quad ; \quad \hat{a}_\rho = \cos\phi\,\hat{a}_x + \sin\phi\,\hat{a}_y$$

$$\hat{a}_y = \sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi \quad ; \quad \hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$$

$$\hat{a}_z = \hat{a}_z \quad ; \quad \hat{a}_z = \hat{a}_z$$

$$A_\rho = A_x\cos\phi + A_y\sin\phi$$

$$A_\phi = -A_x\sin\phi + A_y\cos\phi$$

$$A_z = A_z$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Differential displacement, surface area and volume

a. Differential displacement:

$$dl = d\rho\, a_\rho + \rho\, d\phi\, a_\phi + dz\, a_z$$

b. Differential normal surface area:

$$ds = \rho\, d\phi\, dz\, a_\rho = d\rho\, dz\, a_\phi = \rho\, d\rho\, d\phi\, a_z$$

c. Differential volume:

$$dv = \rho\, d\rho\, d\phi\, dz$$

3. SPHERICAL CO-ORDINATES:

This co – ordinate system is used when we have problem having spherical symmetry.

A point P in this co – ordinate system is represented as, P **(**r**,** θ **,** ϕ**),** and range of values of co – ordinate variable **r,** θ **,** ϕ are

$0 \le r < \infty$	$0 \le \theta \le \pi$	$0 \le \phi < 2\pi$

$r \rightarrow$ is defined as the distance from the origin to point P

OR

the radius of a sphere centered at the origin & passing through P.

$\theta \rightarrow$ is called the colatitude.

It is the angle between the z – axis and the position vector of P.

$\phi \rightarrow$ It is measured from the X - axis.

It is same as azimuthally angle in cylindrical co–ordinates.

Vector $\vec{A}$ in this co – ordinate system is represented as

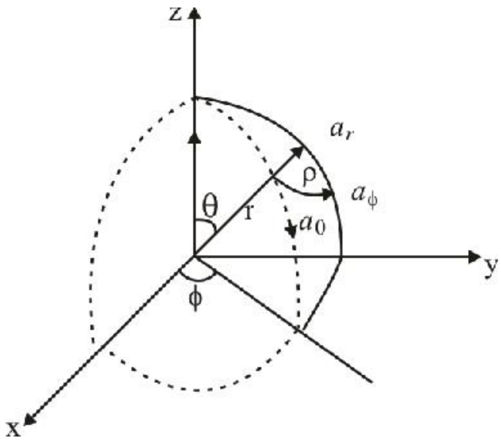
$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi \; ; \text{ Where}$$

a_r, a_θ *and* a_ϕ are unit vector, and a_r directed along the radius or in the direction of increasing r, a_θ in the direction of increasing θ, and a_ϕ in the direction of increasing ϕ .

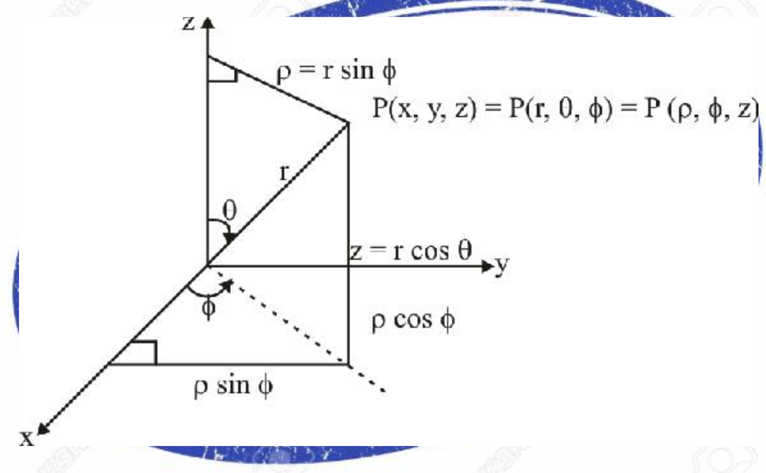
$$a_r . a_r = a_\theta . a_\theta = a_\phi . a_\phi = 1$$

$$a_r . a_\theta = a_\theta . a_\phi = a_\phi . a_r = 0$$

$$a_r \times a_\theta = a_\phi \; ; \; a_\theta \times a_\phi = a_r \; ; \; a_\phi \times a_r = a_\theta$$



Relationship between cylindrical co – ordinates variables and spherical co – ordinates variables:



$$\sin\theta = \frac{\rho}{r} \; ; \; \cos\theta = \frac{z}{r}$$

$$\boxed{\rho = r\sin\theta} \quad ; \quad \boxed{z = r\cos\theta}$$

$$r^2 = \rho^2 + z^2 = x^2 + y^2 + z^2 \qquad \therefore \quad \boxed{r = \sqrt{x^2 + y^2 + z^2}}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\rho}{z} \qquad \boxed{\theta = \tan^{-1}\frac{\rho}{x} = \sqrt{\frac{x^2 + y^2}{z}}}$$

$$\boxed{\phi = \tan^{-1}\frac{y}{x}} \quad ; \quad y = \rho \sin\phi = r\sin\theta \sin\phi$$

$$; \quad x = \rho\cos\phi = r\sin\theta\cos\phi$$

$$; \quad z = r\cos\theta$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Differential displacement, surface area, and volume:

1. Differential displacement:

$$dl = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

2. Differential & normal surface area:

$$ds = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r = r\sin\theta\,dr\,d\phi\,\hat{a}_\theta = r\,dr\,d\theta\,\hat{a}_\phi$$

3. Differential volume:

$$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$$

Note: **distance between two points in different co – ordinate system.**

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \; ; \; cartesian$$

$$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2\cos(\phi_2 - \phi_1) + (z_2 - z_1)^2 \; ; \; cylindrical$$

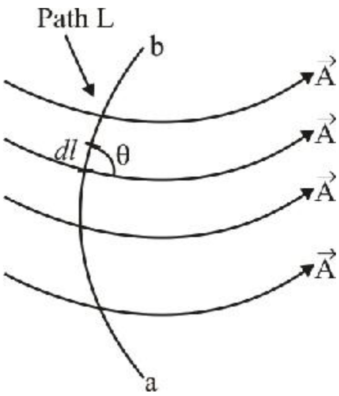
$$d^2 = r_2^2 + r_1^2 - 2r_1 r_2\cos\theta_2\cos\theta_1 - r_1 r_2\sin\theta_2\sin\theta_1\cos(\phi_2 - \phi_1) \; ; \; spherical$$

Line integral, surface integral, volume integral:

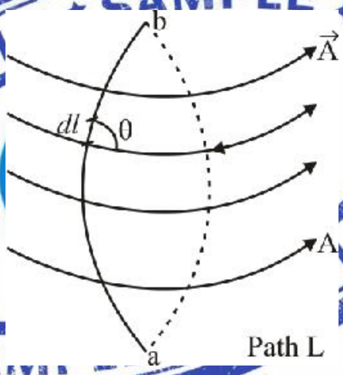
1. Line integral:

It is integral of the tangential component of vector $\vec{A}$ along curve L. and defined as

$$\int_L \vec{A}.\vec{dl} = \int_a^b |A| dl \cos$$



If path of integration is closed curve then it becomes a closed contour integral, which is called the circulation of **A** around **L**. and defined as



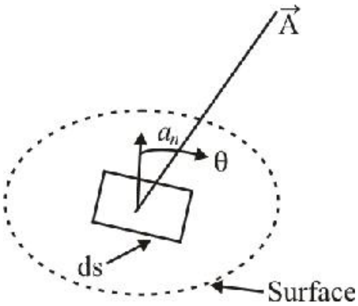
$$\oint_L \vec{A}.\vec{dl}$$

Note: Closed path defines an open surface.

2. Surface integral or flux :

If a vector field $\vec{A}$, is continuous in a region containing the smooth surface S, then flux or surface integral of A through **S** is given by.

$$\cdot = \int_S \vec{A}.\, d\vec{s} = \int_S \vec{A}.\, \hat{a}_n\, ds$$



Where,

$\hat{a}_n$ = unit normal to S.

If surface is closed, then closed surface defines a volume, which is referred as the net outward flux of $\vec{A}$ from, surface S. and defined as,

$$\cdot = \oint_S \vec{A}.\vec{ds}$$

3. Volume integral:

Volume integral is defined as $\int_v \circ_v dv$

Del operator:

- ✓ The Del operator is written as ∇ .
- ✓ It is the vector differential operator.
- ✓ It is also known as gradient operator.

Usefulness of ∇ :

1. The gradient of a scalar V, written as ∇V
2. The divergence of a vector A, written as $\nabla . A$

3. The curl of a vector A, written as $\nabla \times A$

4. The Laplacian of a scalar V, written as $\nabla^2 V$

∇ **In Cartesian co – ordinate system:**

$$\nabla = \hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y} + \hat{a}_z \frac{\partial}{\partial z}$$

∇ **In cylindrical co – ordinate system:**

$$\nabla = \hat{a}_\rho \frac{\partial}{\partial \rho} + \hat{a}_\phi \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{a}_z \frac{\partial}{\partial z}$$

∇ **In spherical co – ordinate system:**

$$\nabla = \hat{a}_r \frac{\partial}{\partial r} + \hat{a}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{a}_\phi \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi}$$

1. **Gradient of a scalar: G**

The gradient of a scalar field V, is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$= \left(\hat{a}_x \frac{\partial V}{\partial x} + \hat{a}_y \frac{\partial V}{\partial y} + \hat{a}_z \frac{\partial V}{\partial z} \right) . \left(\hat{a}_x \, dx + \hat{a}_y \, dy + \hat{a}_z \, dz \right)$$

$$dV = G.dl = |G||dl| \cos\theta$$

$$\frac{dV}{dl} = G \cos\theta \quad ; \quad where \quad G = \hat{a}_x \frac{\partial V}{\partial x} + \hat{a}_y \frac{\partial V}{\partial y} + \hat{a}_z \frac{\partial V}{\partial z}$$

Maximum space rate of change of V occurs when $\theta = 0$

$$\left.\frac{dV}{dl}\right|_{\max} = G = grad\ V$$

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z \ ; \ \text{In Cartesian coordinate}$$

$$\nabla V = \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z \ ; \ \text{In cylindrical coordinates.}$$

$$\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} a_\phi \ ; \ \text{In spherical coordinate.}$$

Computation formulas on gradient:

1. $\nabla (V+U) = \nabla V + \nabla U$
2. $\nabla (VU) = V\nabla U + U\nabla V$
3. $\nabla \left[\frac{V}{U}\right] = \frac{U\nabla V - V\nabla U}{U^2}$
4. $\nabla V^n = nV^{n-1}\nabla V$ Where U & V are scalar field and n is an integer.

Properties of gradient of a scalar field V:

(a) The magnitude of ∇V equals the maximum rates of change in **V** per unit distance.

(b) ∇V Points in the direction of the maximum rate of change in **V**.

(c) ∇V At any point is perpendicular to the constant **V** surface that passes through that point.

(d) The component (*projection*) of ∇V in the direction of a unit vector $\hat{a}$ is $\boxed{\nabla V.\hat{a}}$ and is called directional derivatives of V along $\hat{a}$.

(e) $\boxed{\nabla V . a}$ Is the rate of change of **V** in the direction of **a**.

(f) If $\vec{A} = \nabla V$; then **V** is said to be the scalar potential of **A.**