Chemical Engineering (GATE & PSUs)

Postal Correspondence

GATE & Public Sectors

Heat Transfer

GATE 2015 Top Results

Chemical Engineering

1st Rank

Archhit Trichal

2nd Rank

Keval Pareta

GATE 2015 Result

Name: ARCHHIT TRICHAL

Registration Number: CH88041S1135

Gender: Male

Examination Paper: Chemical Engineering (CH)

Marks out of 100#: 65.67

All India Rank in this paper: 1

Qualifying Marks##: 27.52 (General), 24.77 (OBC (NCL)), 18.34 (SC/ST/PwD)

GATE Score: 947

Highest Result in GATE 2015

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Total 39 Ranks under AIR 100

GATE 2014 Topper

Chemical Engineering

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Chemical Engineering (GATE & PSUs)

GATE 2015 Cut-off Marks				
BRANCH	**GENERAL**	**SC/ST/PD**	**OBC(Non-Creamy)**	**Total Appeared**
Chemical Engineering	27.52	18.34	24.77	15874

Heat Transfer: Conduction, convection and radiation, heat transfer coefficients, steady and unsteady heat conduction, boiling, condensation and evaporation; types of heat exchangers, evaporators and their design.

CONTENT

1. CHAPTER — INTRODUCTION

Heat Transfer: It is the science that seeks to product the energy (Heat) transfer that may take place between material bodies as a result of temperature difference.

- It explains how and at which rate heat energy is transferred between material bodies.
- Any transport takes place due to the presence of driving force.
- In heat transfer, the driving force is thermo potential difference that is also called temperature difference.
- Heat transfer takes place by various modes or mechanisms.
- These mechanisms and the related principles enable us to calculate the rate and extent of transport of thermal energy.
- Heat transfer also explains temperature history w.r.t. time and w.r.t place or co-ordinate.

Difference between Thermodynamics and Heat Transfer:

- Thermodynamics deal with the relation between heat and other forms of energy whereas heat transfer concerned with the analysis of the rate of heat transfer.
- Thermodynamics deals with the systems at equilibrium, heat transfer deals with the system that lack of thermal equilibrium, and hence it is a non-equilibrium phenomenon.
- Thermodynamics is used to find out the amount of thermal energy is needed for changing a system from one equilibrium state to another.
- Thermodynamics doesn't tell anything about the history of the heat transfer rate or temperature while changing from one equilibrium state to another between material bodies.

Modes of Heat Transfer:

1. Conduction 2. Convection 3. Radiation

1. Conduction

- Conduction takes place in solid or stagnant liquid or gaseous medium due to existence of a temperature difference.
- It is not associated with the movement and displacement of particles of the medium from their original position.
- It is achieved by two mechanisms:

(i) Molecular interaction where heat transfer takes place by the kinetic motion or direct impact of molecules.

– Molecules at a high energy level impart energy to adjacent molecules at lower energy levels.

– Conduction energy transfer always exist so long as there is a temperature gradient in a system comprising molecules of a solid, liquid or gas.

(ii) By the drift of 'free' electrons as in the case of metallic solids.

– The metallic alloys have a different concentration of free electrons and their ability to conduct heat energy is directly proportional to the concentration of free electrons in them.

- Heat energy is transferred from one molecule to the adjacent one through molecular vibrations in general solids, drift of free electrons in metals, collisions in gases and liquids etc but in all types, there is no change of original positions of the molecules.

Basic Law of Conduction = Fourier Law

Fourier Law :

- It is also called Joseph Fourier phenomical law. This law is an empiricial law based on observation.
- According to this law the rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, and inversely proportional to the thickness of the layer.

i.e. $$\frac{\dot{Q}_x}{A} \propto \frac{\partial T}{\partial x}$$

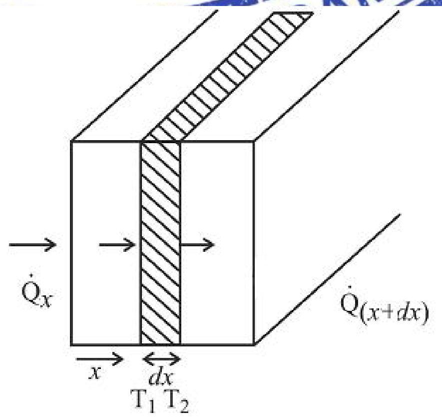
where $\frac{\partial T}{\partial x}$ = Temperature gradient in *x*-direction.

$$\dot{Q}_x = -kA\frac{dT}{dx}$$

where, k = Thermal conductivity $\dot{Q}_x$ = Heat transfer rate

A = Area Unit of k = W/mK k depends on properties of material

- If k changes with position of material = Anisotropic material



- If k does not changes with position of material = Isotropic material.
- For simplicity of the problem, we always assume material is isotropic.
- k can be function of temperature.

$$k = k_o(1 + aT) \qquad k = k_o(1 + aT + bT^2)$$

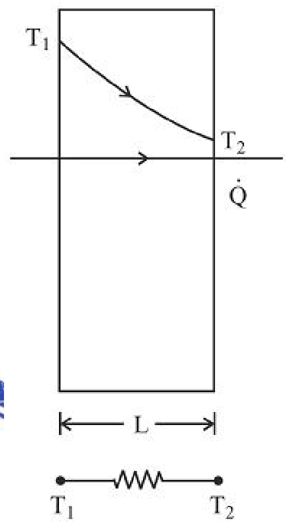
where a and b are constants

- k_{metal} = Very high, k gaseous, non metal = very low.

Assumption of Fourier Law:

1. The heat (energy) flow is in uni-directional.
2. $\frac{dT}{dx}$ = constant and temperature profile linear.
3. There is no internal heat generation.
4. Material is isotropic and homogeneous.
5. Conduction of heat takes place under steady state conditions.

Important Features of Fourier Law:



1. It is useful to all matter *i.e.* solid, liquid, gas.
2. It is a vector expression representing that heat flow is in the direction of decreasing temperatures.
3. It is applicable to define thermal conductivity of medium through which energy is conducted.

$$\text{Heat flow} = \frac{\text{Temperature difference}(d\,T)}{\frac{L}{k\,A}}$$

$$\boxed{\dot{Q}_x = \frac{T_1 - T_2}{\left(\frac{L}{k\,A}\right)} = \frac{\text{Driving force}}{\text{Resistance}}}$$

4. Thermal resistance, $\boxed{(R_{th})_{conduction} = \frac{L}{kA}}$

5. The reciprocal of thermal resistance is known as thermal conductance it is denoted by C.

Conduction in Gases, Liquids and Solids:

- In gases rate of heat transfer by conduction increases with temperature.
- Thermal conductivity (k) of gases increases with increase in temperature.
- Thermal conductivity of gases ranges from 0.006 to 0.6 W/(m.K) while that of liquids ranges from 0.09 to 0.7 W/(m.K).
- K_{metal} varies from 2.3 to 420 W/(m.K).
- The materials having law values of there mal conductivity (less than 0.2 W/(m.K) are called as and used as heat insulators.
- k_{gas} is almost independent of pressure upto a moderate pressure.
- k_{gas} Changes when pressure is critical pressure of gas or more.
- In gases, Hydrogen has highest thermal conductivity of 0.175 W/(m.K) at 0ºC.
- k (thermal conductivity) of water increases with temperature
- k_{liquid} is almost independent of pressure.
- In liquid and gases, conduction takes place due to collisions between molecules of varying temperature.
- In solid conduction, thermal energy conducted by lattice vibration and transport by free electrons.
- Good electrical conductors are also good thermal conductors like metals.

- Good electrical insulators are also good thermal insulators except diamond.
- Diamond is an electrical insulator but thermal conductivity is 5 times as high as Cu and Ag.
- Solid with low k values are thermal insulators. (Ceramic materials, diatomaceous earth, expanded polymeric forms)

2. Convection

It is possible only in presence of a fluid medium.

– If a fluid flows inside a duct or over a solid body and the temperatures of the fluid and the solid surfaces are different, therefore, heat transfer takes between the fluid and solid surface will take place. The carrying of heat here is inseparably linked with the movement of fluid itself.

– Convective heat transfer is associated with displacement of fluid element

– Depending upon type of displacement, it is divided into free/natural convection and forced convection.

Free/Natural Convection

- When the fluid motion set up by buoyancy effects resulting from the density variation caused by the temperature difference in the fluid, hence, heat transfer is said to be free or natural convection.

Forced Convection:

When the fluid motion is artificially generated with the help of an external medium *i.e.* fan or blower.

Newton's law of cooling :

For a fluid flowing at a temperature T_∞ over a surface at a temperature T_s as shown in figure below, and $T_\infty < T_s$.

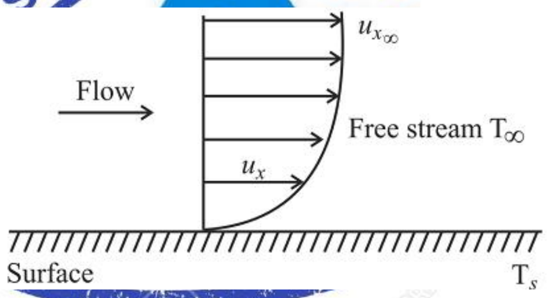


Figure: Convection from a heated pipe

To express the total effect of convection, we can apply Newton's law of cooling:

$$\dot{Q}_x = hA(T_s - T_\infty) = \frac{(T_s - T_\infty)}{\left(\frac{1}{hA}\right)}$$

h = Convective heat transfer coefficient or Film conductance (W/m^2K).

A = Surface Area

$$\dot{Q}_x = \frac{(T_s - T_\infty)}{\frac{1}{hA}} = \frac{\text{Driving force}}{\text{Resistance}}$$

$\dot{Q}_x$ = Rate of heat transfer in watt

- $h = f(\mu, \rho, C_p, k, v)$
- The value of heat transfer coefficient depends upon.

(*i*) Thermodynamic and transport properties, for example-viscosity, density, specific heat etc.

(*ii*) Nature of heat flow

(*iii*) Geometry of the surface

- $(R_{th})_{convection} = \dfrac{1}{hA}$

3. Radiation When two bodies at different temperatures are placed in an evacuated adiabatic enclosure so that they are not in contact through a solid or fluid medium, the temperature of the two bodies will tend to become identical.

- The mechanism or mode of heat transfer by which this equilibrium is obtained is called thermal radiation.
- Radiation heat transfer is an electromagnetic wave phenomenon, and no medium is necessitated for its propagation.
- In fact the heat transfer by radiation is maximum if the two bodies exchanging energy are separated by a perfect vacuum.
- Thermal radiation depens only on the temperature and on the optical properties of the emitter.

Stefan Boltzmann Law:

The maximum rate of radiation that can be emitted from a surface at thermodynamic temperature is

$$\dot{Q}_{emit,max} = \sigma A_s T_s^4 \quad (W)$$

where, σ = Stefan Boltzmann constant = 5.67×10^{-8} W/ $(m^2.k^4)$

A_s = surface Area (m^2) T_s = Absolute temperature of surrounding in K.

The law states that emissive power of black body is directly proportional to fourth power of its absolute temperature.

i.e., $Q \propto T^4$

- $\dot{Q}_{emit,max} = \sigma A_s T_s^4$ (*only applicable for Black Body*)
- For Gray or other body: $\dot{Q}_{net} = f_\varepsilon f_G \sigma A(T_1^4 - T_2^4)$

where, f_ε = Emissivity function f_G = Geometric factor, view factor

T_1, T_2 = Temperature of two different surfaces

Why to Study Heat Transfer in Engineering:

- Almost all industrial processes are involved with heating or cooling, which requires heat transfer.
- For a better process engineer, we should have sound knowledge and understanding of principles, mechanisms and applications of heat transfer.
- It is used for optimal design of heat exchangers.
- The design of chemical plants is done on the basis of heat transfer and the analogous to the mass transfer processes.
- Civil engineers must take care of the thermal effects in buildings and other structures.

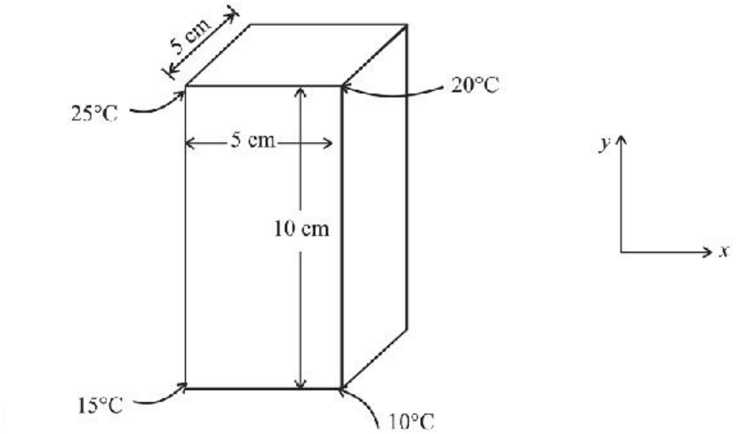
Practice Question (Introduction)

Q. 1. The block of stainless steel shown below is well insulated on the front and back surfaces, and the temperature in the block varies linearly in both the x and y-directions, find:

(a) The heat flux and heat flows in the x and y-directions.

(b) The magnitude and direction of heat flux vector.

Thermal conductivity of material = 14.4 W/(mK).



Solution:

(a) The cross-sectional areas are:

$$A_x = 10\times 5 = 50\,\text{cm}^2 = 0.0050\,\text{m}^2 \qquad A_y = 5\times 5 = 25\,\text{cm}^2 = 0.0025\,\text{m}^2$$

So, the heat fluxes are: $\hat{q}_x = -k\dfrac{\partial T}{\partial x} = -k\dfrac{\Delta T}{\Delta x} = -14.4\dfrac{(-5)}{(0.05)} = 1440\ \text{W/m}^2$

$$\hat{q}_y = -k\frac{\partial T}{\partial y} = -k\frac{\Delta T}{\Delta y} = -14.4\left(\frac{10}{0.1}\right) = -1440\ \text{W/m}^2$$

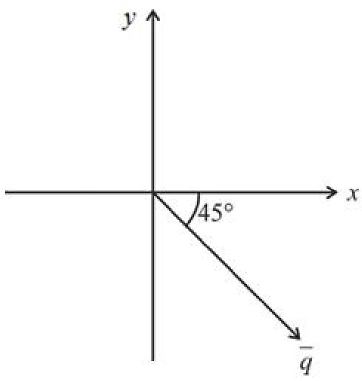
So, the heat flows are: $q_x = \hat{q}_x A_x = 1440\times 0.005 = 7.2\ \text{W}$

$$q_2 = \hat{q}_y A_y = -1440\times 0.0025 = -3.6\ \text{W}$$

(b) $\vec{\hat{q}} = \hat{q}_x\,\vec{i} + \hat{q}_y\,\vec{j}$

$$\vec{\hat{q}} = 1440\,\vec{i} - 1440\,\vec{j}$$

$$|\vec{\hat{q}}| = [(1440)^2 + (-1440)^2]^{0.5} = 2036.5\ \text{W/m}^2$$



The angle θ, between the heat flux vector and the x-axis is calculated as follows:

$$\tan\theta = \frac{\hat{q}_y}{\hat{q}_x} = \frac{-1440}{1440} = -1.0 \qquad \theta = -45^\circ$$

So the directions of heat flow:

Q. 2. The heat flux, q is $5000\ \text{W/m}^2$ at the surface of an electrical heater. The heater temperature is 150º C when it is cooled by air at 50º C. What is the average convective heat transfer coefficient, $\bar{h}$? What will the heater temperature be if the power is reduced so that q is 3000 W/m^2?

Solution : $$\bar{h} = \frac{q}{\Delta T} = \frac{5000}{(150-50)} = 50\ \text{W/(m}^2\text{K)}$$

If the heat flux is reduced, $\bar{h}$ should remain unchanged during forced convection. Thus,

$$\Delta T = T_{heater} - 50^\circ C = \frac{q}{h} = \frac{3000\ \text{W/m}^2}{50\ \text{W/(m}^2.\text{K)}} = 60\ \text{K}$$

So, $$\Delta T = T_{heater} = 60 + 50 = 110^\circ C$$

Q. 3. Consider a person standing in a room temperature maintained at 22º C at all times. The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 5º C in winter and 25º C in summer. Determine the rate of radiation heat transfer between the person and the surrounding surfaces if the exposed surface area and the average outer surface temperature of the person are $1.0\ \text{m}^2$ and 30º C. Assume no convective heat transfer. The emissivity of person $\varepsilon = 0.95$.

Solution : So $$\dot{Q}_{rad,\ winter} = \varepsilon\,\sigma\,A_s(T_s^4 - T^4_{surr,\ winter})$$

$$= 0.95 \times \left(5.67 \times 10^{8}\ \frac{\text{W}}{\text{m}^2\text{K}^4}\right)(1.0\ \text{m}^2) \times [(30+273)^4 - (5+273)^4]\ \text{K}^4 = 132.3\ \text{W}$$

$$\dot{Q}_{rad,\ summer} = \varepsilon\,\sigma\,A_s(T_s^4 - T^4_{surr,\ summer})$$

$$= 0.95 \times \left(5.67 \times 10^{-8}\ \frac{\text{W}}{\text{m}^2\text{K}^4}\right)(1.0\ \text{m}^2) \times [(30+273)^4 - (25+273)^4]\ \text{K}^4 = 29.23\ \text{W}$$

2. CHAPTER

CONDUCTION

One dimensional heat conduction analysis:

$\rightarrow$ Suppose the one dimensional system shown in fig. when the system is in steady state *i.e.*, the temperature does not change with time, and then we need only integrate the equation.

$$Q_{conductance} = -kA\frac{dT}{dx}$$

And substitute the appropriate values to solve for desired quantity.

$\rightarrow$ Let the general case where the temperature may be changing or varying with time and heat sources may be present within the body.

$\rightarrow$ For the element of thickness dx the energy balance may be made.

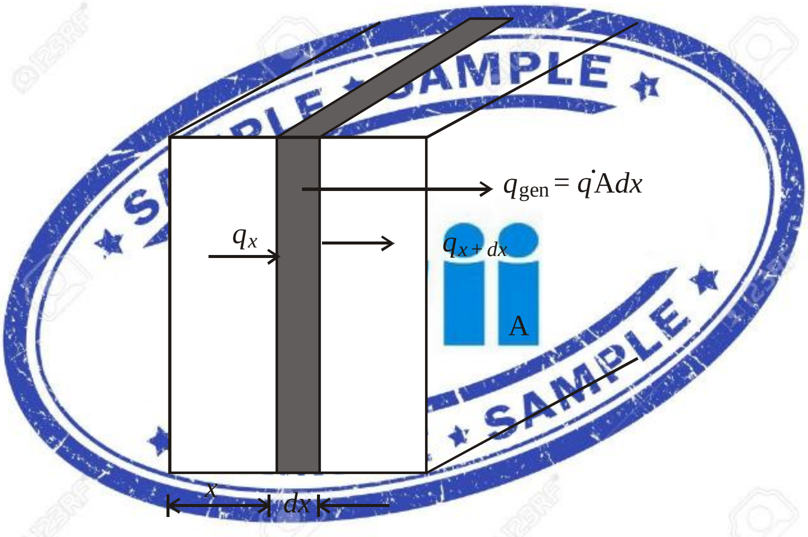


Figure: Elemental volume for one-dimensional heat conduction analysis

Energy conducted in left face + Heat generated within element = Change in internal energy + Energy conducted out right face.

$$q_x + q_{gen} = \rho CA\frac{\partial T}{\partial t}dx + q_{x+dx} \qquad ...(1)$$

As we know.

$$q_x = -kA\frac{\partial T}{\partial x}\ , \quad q_{gen} = \dot{q}\,A\,dx \quad \&\ q_{x+dx} = -kA\left.\frac{\partial T}{\partial x}\right|_{x+dx} = -A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

Now equation (1) becomes,

$$-kA\frac{\partial T}{\partial x} + \dot{q}\,Adx = -A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right] + {}^{\circ}CA\frac{\partial T}{\partial t}dx$$

where, $\quad \dot{q}$ = energy generated per unit volume, W/m^3

C = specific heat of material, J/kg K ρ = density, kg/m^3

Then finally one dimensional equation becomes $\boxed{\dfrac{\partial}{\partial x}\left(k\dfrac{\partial T}{\partial x}\right)+\dot{q}=\rho C\dfrac{\partial T}{\partial t}}$

Heat Conduction Equation in Cartesian Co-ordinates:

To treat more than one-dimensional heat flow, we necessary consider only the heat conducted in and out of a unit volume in three co-ordinate directions, as shown in fig:

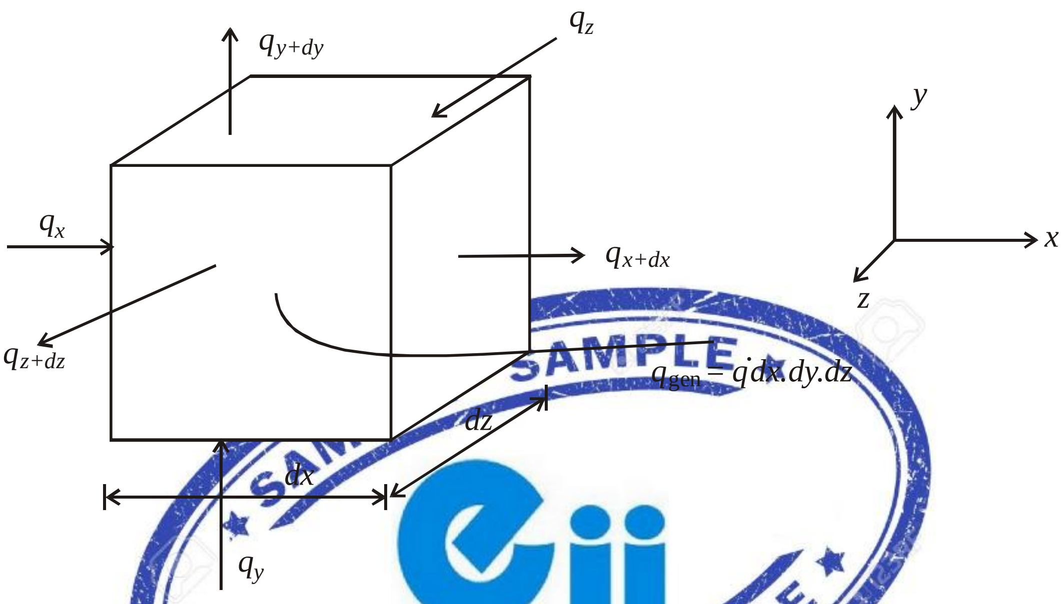


Fig.: Elemental volume for three dimensional heat conduction analyses

Energy Balance

$$q_x + q_y + q_z + q_{gen} = q_{x+dx} + q_{y+qy} + q_{z+dz} + \frac{dE}{dt} \qquad \ldots(i)$$

and the energy quantities are

$$q_x = -k\, dz\, dy\, \frac{\partial T}{\partial x}$$

$$q_{x+dx} = -\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]dz\,.\,dy$$

$$q_y = -k\, dx\, dz\, \frac{\partial T}{\partial y}$$

$$q_{y+dy} = -\left[k\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)dy\right]dx\,.\,dz$$

$$q_z = -k\, dx\, dy\, \frac{\partial T}{\partial z}$$

$$q_{z+dz} = -\left[k\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)dz\right]dx\,dy$$

$$\dot{q}_{gen} = \dot{q}\,dx\,dy\,dz$$

$$\boxed{\frac{dE}{d\tau} = \rho C\,dx\,dy\,dz\,\frac{\partial T}{\partial \tau}}$$

Three-dimensional heat conduction equation is after putting above value in equation (*i*)

$$\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + \dot{q} = \rho C\frac{\partial T}{\partial \tau} \qquad \ldots(ii)$$

For constant thermal conductivity equation (*ii*) becomes:

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho C}{k}\frac{\partial T}{\partial \tau}}$$

Thermal diffusivity = $\dfrac{\text{Thermal conductivity}}{\text{Thermal capacity}}$

$$\alpha = \frac{k}{\rho C}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\overset{o}{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Other Simplified forms of heat conduction equation in Cartesian co-ordinates:

(1) At the situation if temperature does not depend on time, the conduction then takes place in the steady state $\left(i.e. \dfrac{\partial T}{\partial \tau} = 0\right)$ and equation for steady state (called the poisson eq.): $\boxed{\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2} + \dfrac{\dot{q}_g}{k} = 0}$

(2) Transient, no heat generation (called the diffusion equation) : $\boxed{\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}}$

(3) Steady state, no heat generation (Laplace equation): $\boxed{\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2} = 0}$

(4) Unsteady state, one dimensional, without heat generation: $\boxed{\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial \Gamma}}$

(5) Steady state, two dimensional, without heat generation: $\boxed{\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} = 0}$

(6) Steady state, and without heat generation, one dimensional heat transfer: $\boxed{\dfrac{\partial^2 T}{\partial x^2} = 0}$

(7) Steady state and one-dimensional heat transfer: $\boxed{\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\dot{q}_g}{k} = 0}$

General heat conduction equation in cylindrical co-ordinates

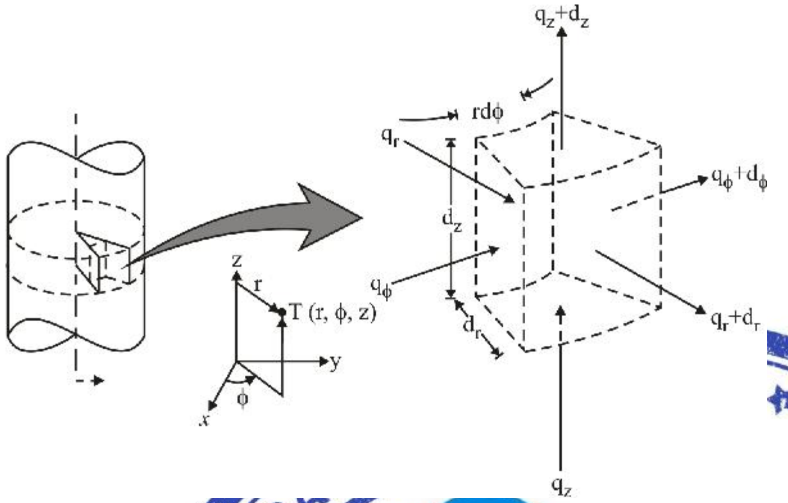


Figure: Differential control volume, $dr.rd\phi.dz$, for conduction analysis in cylindrical coordinates (r, ϕ, z)

$$\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\dot{q}_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Above equation is called the general heat conduction equation in cylindrical co-ordinates.

→ If in case there are no heat sources present and the heat flow is steady and one-dimensional, then

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0 \qquad \frac{1}{r}\frac{\partial}{\partial r}\left(r\,.\,\frac{\partial T}{\partial r}\right) = 0$$

when $\dfrac{1}{r} \neq 0,$ Hence,

$$\frac{\partial}{\partial r}\left(r\,.\,\frac{\partial T}{\partial r}\right) = 0 \quad \text{or} \quad r\frac{\partial T}{\partial r} = \text{constant}$$

General Heat conduction equation in spherical co-ordinate:

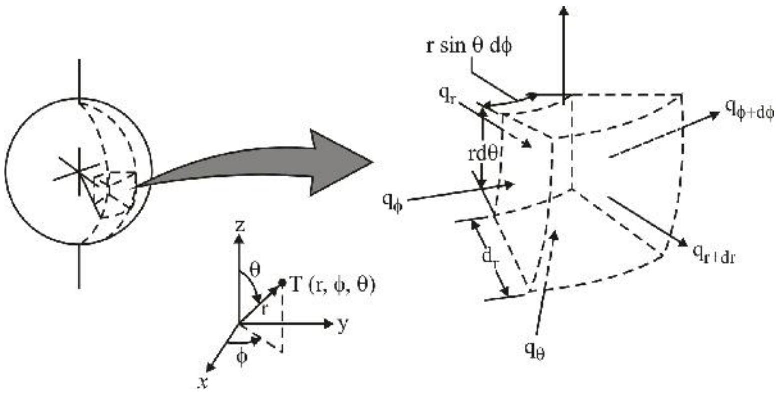


Figure: Differential control volume, $dr.r\sin\theta\, d\phi.rd\theta$, for conduction analysis in spherical co-ordinates (r, ϕ. θ)

$$\left[\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 T}{\partial\phi^2}+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2.\frac{\partial T}{\partial r}\right)\right]+\frac{\dot{q}_g}{k}=\frac{\rho C}{k}\frac{\partial T}{\partial t}$$

$$\left[\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 T}{\partial\phi^2}+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2.\frac{\partial T}{\partial r}\right)\right]+\frac{\dot{q}_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The above equation is the general heat conduction (Spherical co-ordinates).

→ No heat source present, steady and one-dimensional, then the equation becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$