Chemical Engineering (GATE & PSUs)

Postal Correspondence

GATE & Public Sectors

Instrumentation & Process Control

GATE 2015 Top Results

Chemical Engineering

1st Rank
Archhit Trichal

2nd Rank
Keval Pareta

GATE 2015 Result

Name
ARCHHIT TRICHAL

Registration Number
CH88041S1135

Gender
Male

Examination Paper
Chemical Engineering (CH)

Marks out of 100#	65.67	All India Rank in this paper	1
Qualifying Marks##	27.52 (General) / 24.77 (OBC (NCL)) / 18.34 (SC/ST/PwD)	GATE Score	947

Highest Result in GATE 2015

Rank 1, 2, 7, 8.......

Total 39 Ranks under AIR 100

GATE 2014 Topper

Chemical Engineering

1st Rank
Sandeep Kumar

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GATE 2015 Cut-off Marks				
BRANCH	**GENERAL**	**SC/ST/PD**	**OBC(Non-Creamy)**	**Total Appeared**
Chemical Engineering	27.52	18.34	24.77	15874

CONTENT

CHAPTER-1

INTRODUCTION OF LAPLACE TRANSFORM

Need For Process Control

Why process control subject in Chemical Engineering?

- When we run a kinetics experiment, how do we maintain the temperature and level at desired values?
- How do we manufacture products with consistently high quality when raw material properties change?
- How much time do I have to respond to a dangerous situation?

There is a need for continuous monitoring of the operation of a chemical plant and external control to guarantee the satisfaction of operational objectivities.

A control system must:

- Suppress the influence of external disturbances
- Ensure the stability of a chemical process.
- Optimize the performance of a chemical process.

Every engineer needs basic knowledge about control. With this insight, we will be able to design plants that can be controlled safely and produce high quality products

1A Linear differential equation of order n

$$L_n(y) = \frac{d^n y}{dt^n} + A_1(t)\frac{d^{n-1}y}{dt^{n-1}} + ...+ A_n(t)y$$

Linear differential equations arise from mathematical modelling of chemical processes.

Use of Laplace transform

Laplace transform offers a very simple method of solving linear differential equations.

Using Laplace transform, a linear differential equation is reduced to an algebra problem. (which is simpler than solving differential equation directly).

DEFINITION OF LAPLACE TRANSFORM:

The Laplace transform of a function f(t) is defined as f(s) which can be find according to the equation

$$\bar{f}(s) = \int_0^\infty f(t)e^{-st}dt$$

Notation of Laplace transform of f(t) is $\mathcal{L}\{f(t)\} = \bar{f}(s)$

Example 1.1 Laplace transform of function $F(t) = 4$

$$f(s) = \int_0^\infty 4e^{-st}dt = \left[\frac{-4e^{-st}}{s}\right]_0^\infty = \frac{4}{s}$$

$$\mathcal{L}\{4\}=\frac{4}{s}$$

FACTS ABOUT LAPLACE TRANSFORM

1. The Laplace transform is not defined for the function f (t), when the value of '**t**' is less than zero .

2. The Laplace transform is linear. Mathematically $\mathcal{L}\{af_1(t)+bf_2(t)\}=a\mathcal{L}\{f_1(t)\}+b\mathcal{L}\{f_2(t)\}$ Where a and b are constants.

2A

3. Laplace transform of the function f(t) exists if the integral $\int_0^\infty f(t)e^{-st}dt$ takes a finite value (i.e. remains bounded)

4. Laplace transform is a transformation of a function from time domain (where time is independent variable) to s – domain (where S is independent variable)

S is a variable defined in complex plane (i.e. S = a + jb)

LAPLACE TRANSFORMS OF SIMPLE FUNCTIONS

1. **The step function**

$$f(t)=\begin{Bmatrix} 0 & t<0 \\ A & t>0 \end{Bmatrix}$$

$$\mathcal{L}\,[u(t)]=\int_0^\infty (A)e^{-st}dt=\left[\frac{-e^{-st}}{s}\right]_0^\infty A=\frac{A}{s}$$

$$L[A]=\frac{A}{S}$$

3A: L (Step function of size A) = A/S

When function is unit step.

i.e. $u(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$

$$\boxed{L(u(t)=\frac{1}{S}}$$

2. **The exponential function**

f (t) = $\begin{bmatrix} o & t<o \\ e^{-at} & t>o \end{bmatrix}$

$$\mathcal{L}\{e^{-at}\}=\int_0^\infty e^{-(s+a)t}dt=\left[\frac{-1}{s+a}e^{-(s+a)t}\right]_0^\infty$$

$$\mathcal{L}\{e^{-at}\}=\frac{1}{s+a}$$

4A: Similarly,

$$L\left[e^{at}\right]=\frac{1}{S-a}$$

3. The ramp function

$$f(t) = \begin{Bmatrix} o & t<o \\ at & t>o \end{Bmatrix}$$

$$\mathcal{L}\{at(t)\}=\int_0^{\infty} ate^{-st}dt$$

$$\mathcal{L}[at]=\left[-e^{-st}\left(\frac{t}{s}+\frac{1}{s^2}\right)\right]_0^{\infty} a=\frac{a}{s^2}$$

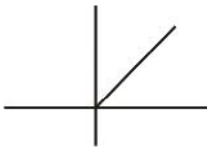
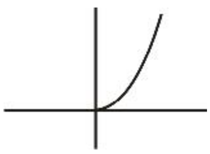
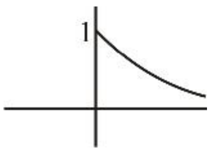
4. <u>The sine function</u>

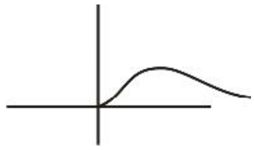
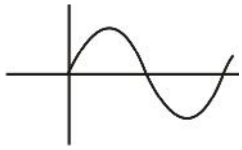
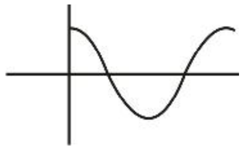
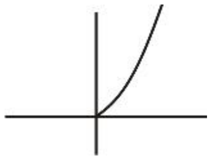
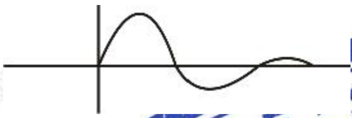
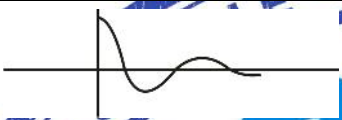
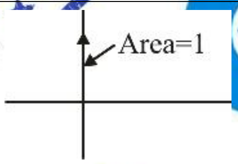
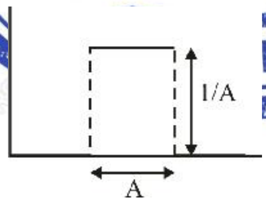
$$F(t) = \begin{Bmatrix} o & t<o \\ sinkt & t>o \end{Bmatrix}$$

$$\mathcal{L}(sinkt)=\int_0^{\infty} sinkt\, e^{-st}dt$$

$$=\left[\frac{-e^{-st}}{s^2+k^2}(s.\sin kt+k\cos kt)\right]_0^{\infty}$$

$$\mathcal{L}\{\sin kt\}=\frac{K}{s^2+K^2}$$

Function	Graph	Transform
u(t)		$\frac{1}{s}$
t u(t)		$\frac{1}{s^2}$
t^n u(t)		$\frac{n!}{s^{n+1}}$
e^{-at} u(t)		$\frac{1}{s+a}$

$t^n e^{-at} u(t)$		$\frac{n!}{(s+a)^{n+1}}$
sin k t u(t)		$\frac{k}{s^2+k^2}$
cos k t u(t)		$\frac{s}{s^2+k^2}$
sinh k t u(t)		$\frac{k}{s^2-k^2}$
$e^{-at}\sin k ? u(r)$		$\frac{k}{(s+a)^2+k^2}$
$e^{-at}\cos k t\, u(t)$		$\frac{s+a}{(s+a)^2+k^2}$
δ(t), unit impulse		1
δ_A (t), unit pulse		$\frac{1}{A}\left(\frac{1-e^{-SA}}{S}\right)$

LAPLACE TRANSFORMS OF DERIVATIVES

a) First order derivative

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\}=s\bar{f}(s)-f(o)$$

b) Second order derivative

$$\mathcal{L}\left\{\frac{d^2f}{dt^2}\right\}=\mathcal{L}\left\{\frac{d}{dt}\left(\frac{df}{dt}\right)\right\}=s\mathcal{L}\left\{\frac{df}{dt}\right\}-\frac{df(t)}{dt}\Big|_{t=0}$$

$$=s\left[sf(s)-f(o)\right]-f^1(o)=s^2\bar{f}(s)-sf(o)-f^1(o)$$

c) nth order derivative

$$\mathcal{L}\left\{\frac{d^n f}{dt^n}\right\}=s^n f(s)-s^{n-1}f(0)-s^{n-2}f^{(1)}(0)............sf^{n-2}(0)-f^{n-1}(0)$$

Example 1.2 Find the Laplace transform of the function

$$\frac{dx^2}{dt^2}+\frac{dx}{dt}+x=1,\quad x(o)=x^1(o)=0$$

Solution : $$\mathcal{L}\left[\frac{dx^2}{dt^2}\right]=s^2x(s)-sx(o)-x^1(o)$$

$$\mathcal{L}\left[\frac{dx}{dt}\right]=sx(s)-x(o)$$

$$\mathcal{L}[x]=x(s)$$

$$\mathcal{L}[1]=\frac{1}{s}$$

We get

$$S^2x(s)-Sx(o)-x^1(o)+Sx(s)-x(o)+x(s)=\frac{1}{S}$$

$$(S^2+S+1)x(s)=\frac{1}{S}$$

$$x(s)=\frac{1}{s(s^2+s+1)}$$

Laplace transform of an integral

if $\mathcal{L}\{f(t)\}=f(s)$,then

$$\mathcal{L}\left\{\int_o^t f(t)dt\right\}=\frac{f(s)}{s}$$

Inversion by partial fraction

In the following example, the technique of partial fraction inversion for solution of differential equation is represented.

Example 1.3 solve the following equation for x(t)

$$\frac{dx}{dt}=\int_o^t x(t)dt-t$$

$$x(o)=3$$

Solution : Taking Laplace transform of above equation

$$\mathcal{L}\left[\frac{dx}{dt}\right]=\mathcal{L}\left[\int_0^t x(t)dt\right]-\mathcal{L}[t]$$

$$sx(s)-x(o)=\frac{x(s)}{s}-\frac{1}{s^2}$$

$$sx(s)-3=\frac{x(s)}{s}-\frac{1}{s^2}$$

$$x(s)=\frac{3s^2-1}{s(s+1)(s-1)}$$

Expanding it by partial fraction method

$$\frac{3s^2-1}{s(s+1)(s-1)}=\frac{A}{s}+\frac{B}{(s+1)}+\frac{C}{(s-1)}$$

$$=\frac{A(s^2-1)+B\{s(s-1)\}+C\{s(s+1)\}}{s(s+1)(s-1)}$$

$$\frac{3s^2-1}{s(s+1)(s-1)}=\frac{s^2(A+B+C)+s(C-B)+(-A)}{s(s+1)(s-1)}$$

Comparing the coefficients on both side

$A+B+C = 3$, $C - B = 0$, $-A = -1$

We get $A = 1, B = 1, C = 1$

$$X(s)=\frac{1}{s}+\frac{1}{s+1}+\frac{1}{s-1}$$

$$X(t)=1+e^{-t}+e^{t}$$

PROPERTIES OF TRANSFORMS

Final value theorem

If f(s) is the Laplace transform of f(t), then

$$\lim_{t\to\infty}\left[f(t)\right]=\lim_{s\to 0}\left[sf(s)\right]$$

Provided that sf(s) does not become infinity for any value of s satisfying Re (s) $\geq$ o. the limit of f(t) is found to be correct only if f(t) is bounded as t approaches infinity

6A The final value theorem allows us to compute the value that a function approaches as $t \to \infty$ when its laplace transform is known.

Example 1.4 Find the final value of the function x(t) for which the Laplace transform is

$$x(s)=\frac{1}{s(s^2+3s^2+6s+8)}$$

Solution apply final value theorem

$$\lim_{t\to\infty}[x(t)]=\lim_{s\to 0}\left[sf(s)\right]=\frac{1}{8}$$

$$\lim_{t\to\infty} [x(t)] = \frac{1}{8}$$

The conditions of the theorem satisfied unless s=–2 or (s+2)$\neq$ 0

Initial value theorem

$$\lim_{t\to 0}\left[f(t)\right] = \lim_{s\to\infty}\left[sf(s)\right]$$

Translation of transform

If $\mathcal{L}\left[f(t)\right] = f(s)$ then

$$\mathcal{L}\{e^{-at}f(t)\} = f(s+a) = \int_0^\infty f(t)e^{-(s+a)t}dt$$

Translation of function

if $\mathcal{L}\left[f(t)\right] = f(s)$ then

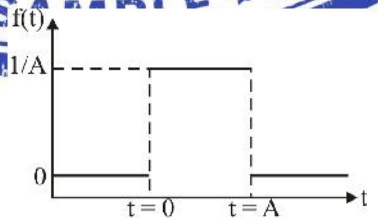
$$\mathcal{L}\left[f(t\text{-}t_0)\right] = e^{-sto}f(s)$$

for t > 0

7A: Unit Pulse Function

Unit pulse function of duration A is defined by.

$$\text{¢}_A(t) = \begin{cases} 0, & t<0 \\ \dfrac{1}{A}, & 0<t<A \\ 0, & t>A \end{cases}$$



Unit pulse function is the difference of two step functions of equal size 1/A.

First step function occurs at t = 0

$$f_1(t) = \begin{cases} 0, & t<0 \\ \dfrac{1}{A}, & t>0 \end{cases}$$

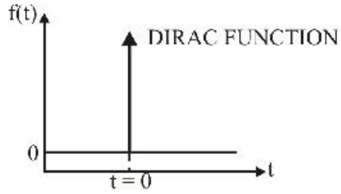
Second step function occurs at t = A

$$f_2(t) = \begin{cases} 0, & t<A \\ \dfrac{1}{A}, & t>A \end{cases}$$

$$\text{¢}_A(t) = f_1(t) - f_2(t)$$
$$\text{¢}_A(t) = f_1(t) - f_1(t-A)$$
$$L[\text{¢}_A(t)] = L[f_1(t)] - e^{-SA} L[f_1(t)]$$
$$= \frac{1}{AS} - e^{-SA}\frac{1}{AS}$$
$$L[\text{¢}_A(t)] = \frac{1}{A}\frac{(1-e^{-SA})}{S}$$

- **Unit Impulse Function**

  As A $\rightarrow$ 0, unit pulse function becomes unit impulse or Dirac function. Represented by δ(t)



$$\int_{-\infty}^{\infty} \text{¢}(t)dt = 1$$
$$\Rightarrow L[\text{¢}(t)] = L\left[\lim_{A\to 0} \text{¢}_A(t)\right]$$
$$= \int_0^{\infty} \lim_{A\to 0} \text{¢}_A(t)e^{-st}dt$$
$$= \lim_{A\to 0}\int_0^{\infty} \text{¢}_A(t)e^{-st}dt$$
$$= \lim_{A\to 0}\left[\frac{1}{A}\frac{1-e^{-SA}}{S}\right]$$

Using L' Hospital rule

$$\lim_{A\to 0}\frac{1}{A}\frac{(1-e^{-SA})}{S} = \lim_{A\to 0}\frac{S\,e^{-SA}}{S} = 1$$
$$\therefore L[\text{¢}(t)] = 1$$

Example 1.5 Solve the following equation for y (t)

$$\int_o^t y(t)dt = \frac{d(y(t))}{dt}, y(o) = 1$$

Solution taking Laplace transforms

$$\mathcal{L}\left[\int_o^t y(t)dt\right] = \mathcal{L}\left[\frac{d(y(t))}{dt}\right]$$
$$\frac{1}{s}y(s) = sy(s) - y(o)$$
$$y(s) = \frac{s}{s^2-1}$$

$$y(t)=\mathcal{L}^{-1}[y(s)]=\mathcal{L}^{-1}\left[\frac{s}{s^2-1}\right]=\cosh t$$

KEY POINTS

1. Laplace transform of a function f (t)

$$f(s)=\int_o^\infty f(t)e^{-st}dt$$

2. t < o ; Laplace transform is not defined

3. $\mathcal{L}\{af_1(t)=bf_2(t)\}=a\mathcal{L}\{f_1(t)\}+b\mathcal{L}\{f_2(t)\}$

4. $\mathcal{L}\left\{\frac{df(t)}{dt}\right\}=sf(s)-f(o)$

5. $\mathcal{L}\left\{\frac{d^2f(t)}{dt^2}\right\}=s^2f(s)-sf(0)-f'(0)$

6. For n^{th} order

$$\mathcal{L}\left\{\frac{d^nf(t)}{dt^n}\right\}=s^nf(s)-s^{(n-1)}f(o)-s^{(n-2)}f^1(o)\ldots\ldots\ldots\ldots\ldots\; sf^{(n-2)}(o)-f^{(n-1)}(o)$$

7. $\mathcal{L}\left\{\int_o^t f(t)\,dt\right\}=\frac{f(s)}{s}$

8. Final value theorem ; $\lim_{t\to\infty}[f(t)]=\lim_{s\to 0}[s\,f(s)]$

9. Initial value theorem

$$\lim_{t\to 0}[f(t)]=\lim_{S\to\infty}[Sf(s)]$$

10. If $\mathcal{L}[f(t)] = f(s)$

Then $\mathcal{L}\{e^{(-at)}f(t)\}=f(s+a)=\int_0^\infty f(t)e^{-(s+a)t}dt$

$\mathcal{L}[f(t-t_o)]=e^{-st_o}f(s)$

Table of Laplace Transforms

Table of Laplace Transform

1) L (1) = 1/S

2) $L(e^{at})=\frac{1}{S-a}$

3) $L(t^n) = \dfrac{n!}{S^{n+1}}$ when t > 0 & n ∈ N

4) $L(t^n) = \dfrac{\sqrt{n+1}}{S^{n+1}}$ where n ∉ N n is function

5) $L(\sin at)\dfrac{a}{S^2 + a^2}$

6) $L(\cos at) = \dfrac{S}{S^2 + a^2}$

7) $L(\sinh at) = \dfrac{a}{S^2 - a^2}$

8) $L(\cosh at) = \dfrac{S}{S^2 - a^2}$

9) $L(e^{at}\ t^n) = \dfrac{n!}{(S-a)^{n+1}}$

CHAPTER-2

TRANSFER FUNCTION

MERCURY THERMOMETER We will develop the transfer function for a first order system by considering the unsteady state behavior of an ordinary mercury glass thermometer

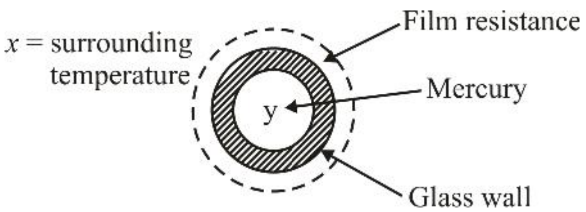


Figure-2.1: cross sectional view of thermometer.

Consider the thermometer which is located in a flowing steam of fluid for which the temperature x varies with time . We need to calculate the time variation of thermometer reading y for a particular change in x .

ASSUMPTIONS

1. The film surrounding the bulb only governs the resistance to heat transfer. Therefore the resistance offered by glass and the mercury is neglected.
2. There is no expansion or contraction of glass during the transient response.
3. The temperature of mercury is uniform throughout

Initially thermo meter is at steady state. At Time the zero the thermometer will be subjected to some change in the surrounding temperature x (t)

By Applying the energy balance

Input rate – output rate = rate of accumulation

$$h\,A(x-y)-0=mC\frac{dy}{dt} \qquad (2.1)$$

Where

A = surface area of bulb for heat transfer

C = heat capacity of mercury

m = mass of mercury in bulb

t = time

h = heat transfer coefficient of film

At steady state

$$hA(x_s - y_s) = 0 \qquad (2.2)$$

The subscript's is used to indicate the variable in the steady state value

Subtracting equation (2.2) from equation (2.1) gives

$$hA[(x-x_s)\ -\ (y-y_s)] = mc\frac{d(y-ys)}{dt}$$

Notice $\frac{d(y-y_s)}{dt} = \frac{dY}{dt}$ because y_s is constant

Put $X = x-x_s$, $Y = y-y_s$

$$hA[X-Y] = mC\frac{dy}{dt}$$

$$[X-Y] = \frac{mc}{hA}\frac{dY}{dt} \qquad \left[put\ \frac{mc}{ha} = \tau\right]$$

$$X-Y = \tau\frac{dY}{dt}$$

Taking Laplace transform

$$X(s) - Y(s) = \tau SY(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau s+1}$$

The parameter τ is called the time constants of the system and has the units of time

$$\frac{Y(s)}{X(s)} = G(s) = \text{transfer function}$$

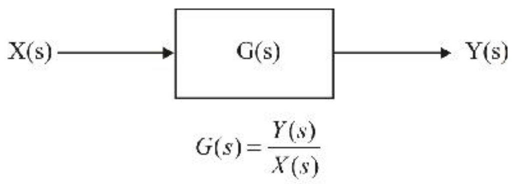
Where G (s) is the symbol for the transfer function of the system

PROPERTIES OF TRANSFER FUNCTION

1. It describe the dynamic behavior of the system

2. It is ratio of the Laplace transform of the deviation in output variable to the Laplace transform of the deviation in the input variable

3. In mercury thermometer output variable is the thermometer reading and input variable is the surrounding temperature



Block diagram representation