SAMPLE STUDY MATERIAL

Postal Correspondence Course

GATE, IES & PSUs

Civil Engineering

Strength of Material

SOM

ENGINEERS INSTITUTE OF INDIA

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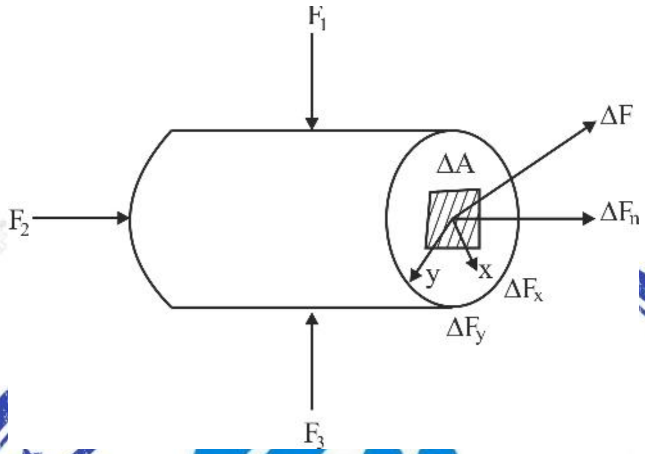
CONTENT

CHAPTER-1

SIMPLE STRESSES AND STRAINS

STRESS (±):

It is the internal resistance offered by a body against the deformation numerically, it is given as force per unit area.



Stress on elementary area ΔA,

i.e. $$\pm = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA} (N/m^2)$$ This unit is called Pa(Pascal)

In case of normal stress *d*F always ⊥ (perpendicular) to area *d*A.

Pascal is a small unit in practice. These units are generally used

$$1\text{kPa} = 10^3 \text{ Pa} = 10^3 \text{ N/ m}^2$$

$$1\text{MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/ m}^2$$

$$1\text{GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/ m}^2$$

1. **Normal Stress:** It may be tensile or compressive depending upon the force acting on the material.

   Tensile and compressive stresses are called **direct stresses.**

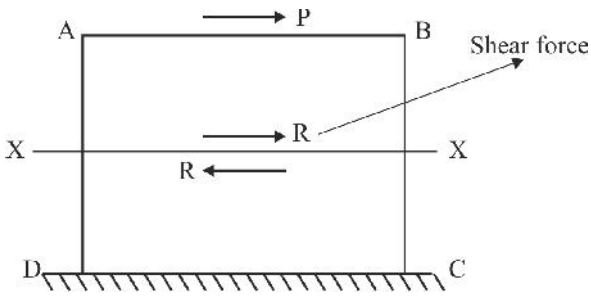
   When, $\sigma > 0$, Tensile

   When, $\sigma < 0$, Compressive

2. **Shear Stress (²):** It is the intensity of shear resistance along a surface (Let X-X).

$$2 = \frac{Shear\ \ force}{Shear\ \ Area}(N/m^2)$$

In case of shear stress force always parallel to the sheared area *i.e.* P is parallel to sheared area in figure.



3. **Conventional or Engineering Stress ($\pm_0$):** It is defined as the ratio of load (P) to the original area of cross-section (A_0):

$$\therefore \quad \pm_0 = \frac{P}{A_0}$$

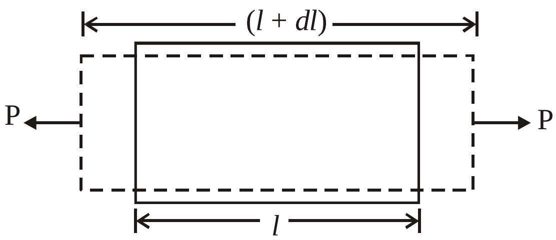
4. **True Stress (±):** It is defined as the ratio of load (P) to the instantaneous area of cross-section (A):

$\therefore \quad \pm = \frac{P}{A}$ *or*, $\pm = \pm_0(1+£)$ Where ε = strain $[Al = A_0 l_0]$ $l = l_0(1+£)$ Initial volume = Final volume

STRAINS (£):

It is defined as the change in length per unit length. It is a dimensionless quantity.

i.e. $$£ = \frac{\text{change in length}}{\text{original length}} = \frac{dl}{l}$$



1. **Conventional or Engineering strain:** It is defined as the change in length per unit original length.

$$£ = \frac{l - l_0}{l_0}$$

Where,

l = Deformed length

l_0 = Original length

e.g. from above figure.

$$\varepsilon = \frac{l + dl - l}{l} \qquad \varepsilon = \frac{dl}{l}$$

2. **Natural Strain:** It is defined as the change in length per unit instantaneous length.

$$\bar{£} = \int_{l_0}^{l} \frac{dl}{l} = ln\frac{l}{l_0} = ln(1 + £) == \ln\left(\frac{A_0}{A}\right) = 2\ln\left(\frac{d_0}{d}\right)$$

Also, $\because \quad \bar{\varepsilon} = \ln(1 + \varepsilon)$

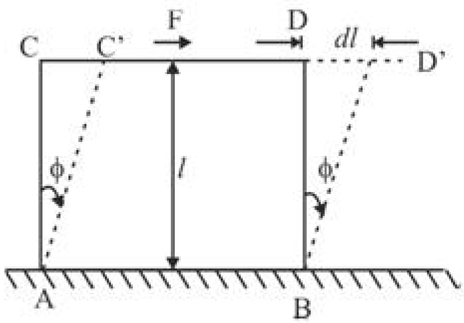
$\Rightarrow \quad 1 + \varepsilon = e^{\bar{\varepsilon}}$

$\Rightarrow \quad \varepsilon = e^{\bar{\varepsilon}} - 1$

Volume of the specimen is a assumed to be constant during plastic deformation

$\because \quad A_0 L_0 = AL$ -Valid till neck formation.

3. **Shear Strain (¤):** It is the strain produced under the action of shear stresses.



Shear Strain = tan ϕ

For small strain, $\tan ¤ \approx ¤$

From figure, $\Delta ACC'$ or $\Delta BDD'$

$$\tan\phi = \frac{dl}{l} = \frac{CC'}{l}$$

$$¤ = \frac{dl}{l} = \frac{\text{Transverse displacement}}{\text{Distance from lower face}}$$

- Shear strain cause deformation in shape but volume remains same.

4. **Superficial strain ($£_s$):** It is defined as the change in area of cross section per unit original area.

$$£_s = \frac{A - A_0}{A_0}$$

Where, A = Final area

A_0 = Original area

5. **Volumetric Strain ($£_v$):** It is defined as the change in volume per unit original volume.

$$£_V = \frac{V - V_0}{V_0}$$

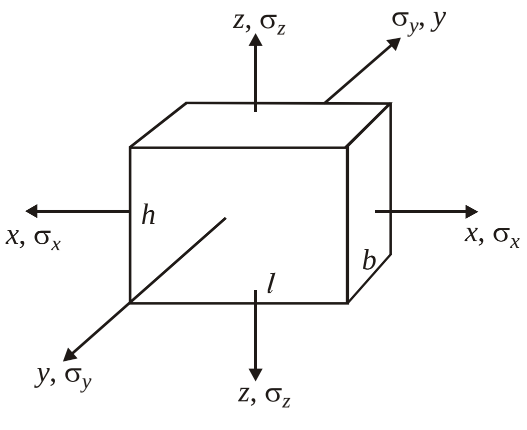
Where, V = Final volume

V_0 = Original volume

- Stress and strain are tensor (*neither vector nor scalar*) of 2nd order.

$$\text{Volumetric strain } \varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Volumetric strain for various shapes



(*i*) Rectangular body:

V = *lbh* on partial differentiation

$$\delta V = \delta l(b.h) + \delta b(l.h) + \delta h(b.l)$$

$$\varepsilon_V = \frac{\delta V}{V} = \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta h}{h}$$

$$\boxed{\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

Note: $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are the strain corresponding to the stresses $\sigma_x, \sigma_y, \sigma_z$ in *x*-direction, *y*-direction, *z*-direction respectively

$$\boxed{\varepsilon_V = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\upsilon)}$$

$\upsilon \rightarrow$ POISSON Ratio

$\upsilon = 0.5$ For rubber

(*ii*) For cylindrical body:

$$V = \frac{\pi}{4} d^2 l$$

$$\delta V = 2\,dl\,.\,\delta d.\frac{\pi}{4} + \frac{\pi}{4} d^2 \delta l$$

$$\varepsilon_V = \frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$

$$\boxed{\varepsilon_V = 2\varepsilon_d + \varepsilon_l}$$

(*iii*) For spherical body

$$\boxed{\varepsilon_V = 3\frac{\delta d}{d}} \qquad \boxed{V = \frac{4}{3}\pi r^3} \qquad d = 2r$$

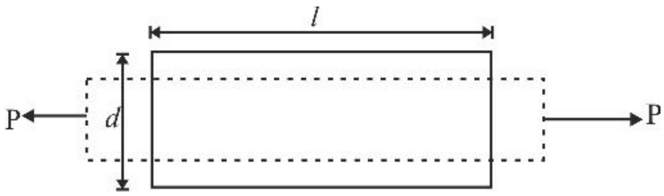
Gauge Length: It is that portion of the test specimen over which extension or deformation is measured.

i.e. this length is used in calculating strain value.

Poisson's ratio $\left(3 \text{ or } \frac{1}{m}\right)$: Value of μ varies between (–1 to 0.5)

The ratio of the lateral strain to longitudinal strain is called the Poisson's ratio.

$$\boxed{\upsilon = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}} \quad \text{or} \quad \boxed{\upsilon = \frac{-\left(\frac{\delta d}{d}\right)}{\left(\frac{\delta l}{l}\right)}}$$



- For a given material, the value of ‘υ’ is constant throughout the linearly elastic range.
- For most of the metals the value of ‘υ’ lie between 0.25 – 0.42
- ‘ 3 ’ varies from (– to 0.5)

Note: ‘υ’ for ductile material is greater than ‘υ’ for brittle metals.

Table

Material	Value of ‘ 3 ’	Remarks
Cork	0	∴ Used in bottle to withstand pressure
Foam	–1	
Rubber	0.5	
Concrete	0.1 – 0.2	
C.I.	0.23 – 0.27	

For cork $v = 0$

For rubber $v = 0.5$

For concrete $v = 0.1 - 0.2$

Isotropic Material: These materials have same elastic properties in all directions.

No. of independent elastic constants = 2, *i.e.* if any of 2 elastic constants is known then other can be derived.

Anisotropic materials: These materials don’t have same elastic properties in all directions.

Elastic modulii will vary with additional stresses appearing. ∵ There is a coupling between shear stress and normal stress for an isotropic material.

Hooke’s Law: It states that when a material is loaded such that the intensity of stress is within a certain limit, the ratio of the intensity of stress to the corresponding strain is a constant which is characteristics of that material.

i.e. $\frac{\text{Stress}}{\text{Strain}} = \text{Constant} = E$ *i.e.,* $\sigma = E\varepsilon$

Where, E = Young's Modulus (N/m^2)

Or

Modulus of Elasticity

- For steel, value of E = 210 GPa (1 GPa = 10^3 N/m^2)
- For aluminum, value of E = 73 GPa $E_{Al} \simeq \frac{1}{3}$rd E_{steel}
- For Plastic, value of E = 1 GPa – 14 GPa

Note : As flexibility increases, value of young's modulus decreases.

It is resistance to elastic strain.

Shear Modulus of Elasticity OR Modulus of Rigidity (G or C): It is defined as the ratio of shearing stress to shearing strain.

$$G \text{ or } C = \frac{\text{Shear stress}}{\text{Shear strain}}$$ *i.e.* ² = G¤

Bulk Modulus (K):

It is defined as the ratio of uniform stress intensity to volumetric strain within the elastic limits.

$$K = \frac{\text{Stress}}{\text{Volumetric Strain}}$$

Note: Elastic constant relationship

(i) $E = 2C(1+v)$, where, v = Poisson's ratio.

(ii) $E = 3K(1-2v)$

(iii) $v = \frac{3K-2C}{6K+2C}$

(iv) $E = \frac{9KC}{3K+C}$

STRESS-STRAIN DIAGRAM:

1. Ductile material (Mild Steel):

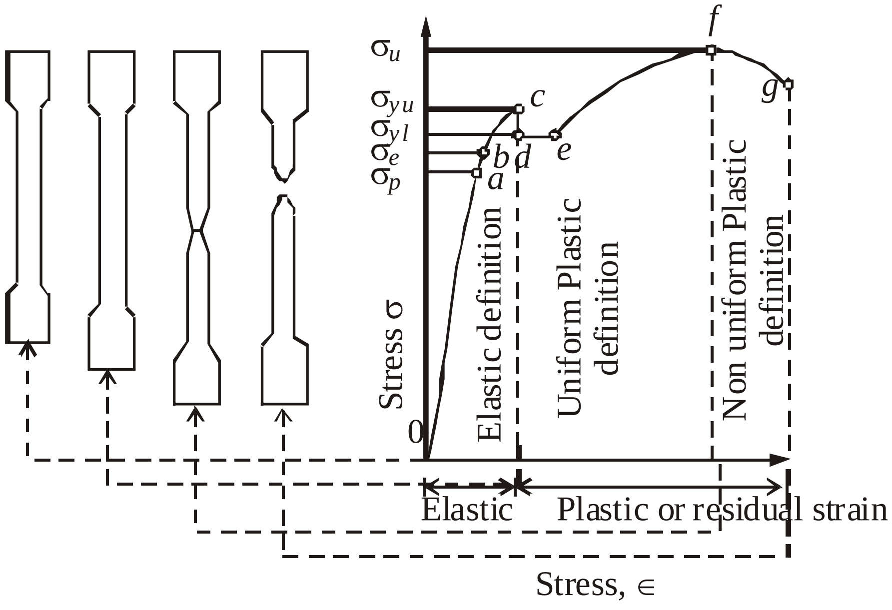


Figure: ***Typical stress-strain diagram for a ductile material***

- Point '*a*' → Limit of proportionality: Up to this point '*a*', Hooke's law is obeyed; '*oa*' is a straight line. Stress corresponding to this point is called 'proportional limit stress, σ_p'

Comparison of Engineering and true stress strain curve:

- The true stress-strain curve is also known as **flow curve.**
- True stress-strain curve gives a true indication of deformation characteristics because it is **based on the instantaneous dimension of specimen.**
- In engineering stress-strain curve, the stress drops down after necking since it is **based on the original area.**
- In true stress strain curve, the stress however increases after necking since the cross section area of the specimen **decreases rapidly after necking.**
- The flow curve of many metals in the region of uniform plastic deformation can be expressed by **simple power law.**

$$\sigma_T = K(\in_T)^n$$

where, K is the strength co-efficient, σ_T is time stress.

n is the strain hardening coefficient.

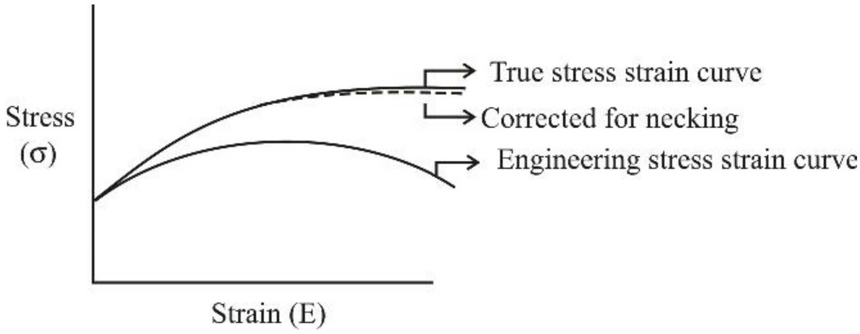
n = 0 for perfectly plastic solid

n = 1 In elastic solid

For most metals 0.1 < *n* 0.5

$\sigma_{True} > \sigma_{Nominal} \rightarrow$ if force is tensile, since area decreases.

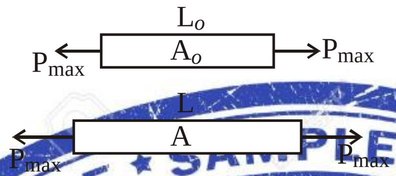
$\sigma_{True} > \sigma_{Nominal} \rightarrow$ if force is compressive, since area increase.



Relation between ultimate tensile strength and true stress at maximum load.

Ultimate tensile strength $\sigma_u = \frac{P_{max}}{A_o}$

True stress at maximum load = $(\sigma_u)_T = \frac{P_{max}}{A}$



True strain at max load $(\varepsilon_T) = \ln\frac{A_o}{A}$ or $\frac{A_o}{A} = e^{\epsilon_T}$

Eliminating P_{max} we get

$$(\sigma_u)_T = \frac{P_{max}}{A} \times \frac{A_o}{A_o}$$

$$= \frac{P_{max}}{A_o} \times e^{\epsilon_T}$$

$$\Rightarrow (\sigma_u)_T = \sigma_u \, e^{\epsilon_T}$$

Here, P_{max} is the max force.

A_o = original cross section area

A = instantaneous cross section area

- Based on the above theory two examples has been provided.

Example 1. Only elongation no neck formation.

In the tension test of rod shown initially it was $A_o = 50\,\text{mm}^2$ and

$L_o = 100\,\text{mm}$. After the application of load its $A = 40\,\text{mm}^2$ and L = 125 mm.

Determine the true strain using changes in both length and area.

Solution: Here $A_o L_o = AL$

i.e., $50 \times 100 = 40 \times 125$

$\Rightarrow$ $5000 \text{ mm}^2 = 5000 \text{ mm}^2$ $\therefore$ no neck formation.

$\therefore$ true strain can be calculated both by area and length formula as follows.

$$\in_T = \int_{l_o}^{l} \frac{dl}{l} = \ln\left(\frac{125}{100}\right) = 0.223$$

$$\in_T = \int_{A_o}^{A} \ln\left(\frac{A_o}{A}\right) = \ln\left(\frac{50}{40}\right) = 0.223$$

II. Elongation with neck formation

Example. A ductile material is tested such that necking occurs then the final gauge length is L = 140 mm and the final minimum cross section area is $A = 35 \text{ mm}^2$ though the rod shown initially was of area $A_o = 50 \text{ mm}^2$ and $L_o = 100 \text{ mm}$. Determine the true strain using change in both length and area.

Solution. Check $A_o L_o = 50 \times 100 = 5000 \text{ mm}^3$

$AL = 35 \times 140 = 4900 \text{ mm}^3$

i.e. $A_o L_o > AL$ $\therefore$ Necking occurs and force applied is tensile.

$$\therefore \quad \in_T = \ln\left(\frac{A_o}{A}\right) = \ln\left(\frac{50}{35}\right) = 0.357$$

$$\in_T = \int_{l_o}^{l} \frac{dl}{l} = \ln\left(\frac{140}{100}\right) = 0.336 \text{ (wrong)}$$

Inference: After necking gauge length gives error but area and diameter can be used for the calculation of true strain at and before fracture.

- Point 'b'$\rightarrow$ Elastic limit point: '*ab*' is not a straight line but upto point '*b*' the material remains elastic. Stress corresponding to this point is called elastic limit stress, σ_e.

  Elastic limit > Proportional limit.

Generally, point '*a*' and '*b*' coincides.

- Point 'c'$\rightarrow$ upper yield point: At this point the cross-sectional area starts decreasing.

- Point 'd'→ Lower yield point: At this point the specimen elongates by a considerable amount without any increase in stress. The value of stress at this point is

  $\pm_y = 250 N/mm^2$ for mild steel.

  The value of strain at yield stress is about 0.0012 or 0.12%

  Lower yield point 'd' is observed, if rate of loading is slow.
- Upper yield point 'c' is observed, if rate of loading is fast.
- Portion '*de*' represents 'plastic yielding': -Typical value of strain is 0.014 or 1.4% i.e. strain in range '*de*' is at least 10 times the strain at the yield point.
- Portion '*ef*' represents 'strain hardening': Strain increases fast with strain, till the ultimate load is reached.
- Point '*f*' → Ultimate stress: Corresponding strain is 20% for mild steel. It is the maximum stress to which the material can be subjected in a simple tensile test. At this point necking of material begins.
- Point 'g' → Breaking Stress: - Corresponding strain is called fracture strain. It is about 25% for mild steal .

Concept of reduced area (RA): $q = \dfrac{A_f - A_o}{A_o}$

- Reduction of area is more a measure of deformation required to produce failure and its chief contribution results from necking process.
- There is a complicate state of stress in necking condition.
- RA is the most sensitive ductility parameter and is useful in detecting quality changes in materials.

2. Brittle Material (Cast Iron):

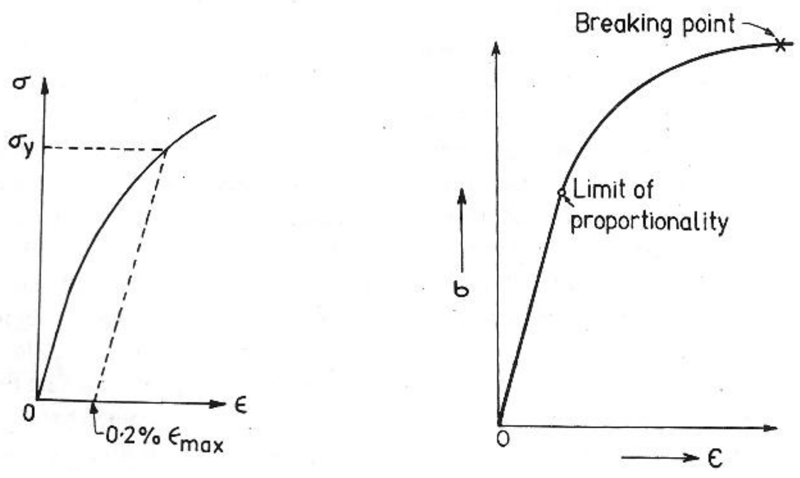


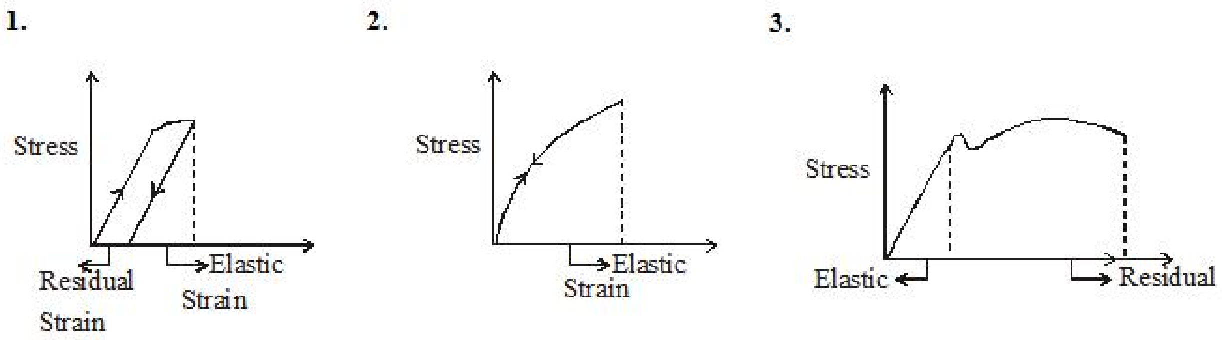
Figure: ***Typical stress-strain diagram for a ductile material***

- In these materials, elongation and reduction in area of the specimen is very small.

- The yield point is not marked at all.
- The straight portion of the diagram is very small.
- **Proof stress:** It is given corresponding to 0.2% of strain. A line parallel to linear portion of curve is drawn passing through 0.2% strain:

$$\boxed{\sigma = \epsilon_{Total}\, E - \epsilon_{Plastic}\, E = \epsilon_{Elastic} \times E}$$

Concept of Elastic and Plastic strain by graph:



PROPERTIES OF METALS:

1. **Ductility:** It is the characteristics of metal by virtue of which, it can be stretched. Large deformations are thus possible in these materials before the rupture takes place.

   e.g. - Mild Steel, Aluminium, Copper, Silver, Gold, Lead etc.

   - Yield failure occurs in ductile materials.