SAMPLE STUDY MATERIAL

Instrumentation Engineering
IN

Postal Correspondence Course
GATE & PSUs
Control System

28-B/7, Jia Sarai, Near IIT, Hauz Khas, New Delhi-110016. Ph. 011-26514888. www.engineersinstitute.com

A Team of IES & GATE Toppers

CONTENT

CHAPTER-1

LAPLACE TRANSFORM

Let F be a function it's Laplace transform is denoted by $\sim(f)$. The Laplace transform f(s) of a function f(t) is defined by

$$\sim(f) = f(s) = \int_0^{\infty} e^{-ts} f(t)dt$$

The integral is evaluated with respect to t , hence once the limits are substituted what is left are is term of s.

Derivation of Laplace Transform:

(1) $$\sim(e^{at}) = \int_0^{\infty} e^{at} e^{-st} dt = \frac{1}{s-a}$$

(2) $$\sim^{-1}\left(\frac{1}{s-a}\right) = e^{at}$$

(3) f(t) = e^{at} put a = jω

$$L\left(e^{j\mu}\right) = \frac{1}{s - j\mu}$$

$$e^{j\mu} = \cos\mu t + j\sin\mu t$$

$$L(\cos\mu t + j\sin\mu t) = \frac{1}{s - j\mu} = \frac{s + j\mu}{s^2 + \mu^2}$$

$$L(\cos\mu t + j\mu t) = \frac{s + j\mu}{s^2 + \mu^2}$$

$$L\left(\cos\mu t\right) = \frac{s}{s^2 + \mu^2} \qquad L\left(\sin\mu t\right) = \frac{\mu}{s^2 + \mu^2}$$

$$L^{-1}\left(\frac{s}{s^2 + \mu^2}\right) = \cos\mu t \qquad L^{-1}\left(\frac{\mu}{s^2 + \mu^2}\right) = \sin\mu t$$

(4) In the function f(t) = e^{at}

$$a = -\sim + j\mu$$

$$e^{at} = e^{(-\sim + j\mu)t} \qquad f(t) = e^{(-\sim + j\mu)t}$$

$$L\left(e^{(-\sim + j\mu)t}\right) = \frac{1}{s - (-\sim + j\mu)} = \frac{1}{(s + \sim) - j\mu}$$

$$e^{(-\sim+j\mu)t} = e^{-\sim t}(\cos\mu t + j\sin\mu t)$$

$$Le^{-\sim t}(\cos\mu t + j\sin\mu t) = \frac{1}{(s+\sim)-j\mu} = \frac{(s+\sim)+j\mu}{(s+\sim)^2+\mu^2}$$

$$L\left(e^{-\sim t}\cos\mu t\right) = \frac{s+\sim}{(s+\sim)+\mu^2} \qquad Le^{-\sim t}\left(\sin\mu t\right) = \frac{\mu}{(s+\sim)^2+\mu^2}$$

$$L^{-1}\left(\frac{s+\sim}{(s+\sim)^2+\mu^2}\right) = e^{-\sim t}\cos\mu t \qquad L^{-1}\left(\frac{\mu}{(s+\sim)^2+\mu^2}\right) = e^{-\sim t}\sin\mu t$$

(5) In the function $f(t) = e^{at}$ put a = 1

$f(t) = e^{t}$

$$L\left(e^{t}\right) = \frac{1}{s-1}$$

Table of L.T

	f(t)	f(s) = L{f(t)}
(1)	f(t) unit impulse at t = 0	1
(2)	u(t) unit step at t = 0	$\frac{1}{s}$
(3)	u(t – T) unit step at t = T	$\frac{1}{s}e^{-sT}$
(4)	t	$\frac{1}{s^2}$
(5)	$\frac{t^2}{2}$	$\frac{1}{s^3}$
(6)	t^n	$\frac{\lfloor n}{s^{n+1}}$
(7)	e^{-at}	$\frac{1}{s+a}$
(8)	e^{at}	$\frac{1}{s-a}$
(9)	te^{-at}	$\frac{1}{(s+a)^2}$
(10)	te^{at}	$\frac{1}{(s-a)^2}$

(11) $t^n e^{-at}$ $\quad \dfrac{\lfloor n}{(s+a)^{n+1}}$

(12) $\sin \omega t$ $\quad \dfrac{\mu}{s^2+\mu^2}$

(13) $\cos \omega t$ $\quad \dfrac{s}{s^2+\mu^2}$

(14) $e^{-\alpha t} \sin \omega t$ $\quad \dfrac{\mu}{(s+\sim)^2+\mu^2}$

(15) $e^{-\alpha t} \cos \omega t$ $\quad \dfrac{s+\sim}{(s+\sim)^2+\mu^2}$

(16) $\sinh \sim t$ $\quad \dfrac{\sim}{s^2-\sim^2}$

(17) $\cosh \sim t$ $\quad \dfrac{s}{s^2-a^2}$

Basic Laplace transform theorems:-

(1) LT of linear combination $L\{af_1(t)+bf_2(t)\} = aF_1(s)+bF_2(s)$

(2) If the Laplace transform of f(t) is F(s) then

(i) $L\left(\dfrac{df(t)}{dt}\right) = sF(s) - f(0^+)$

(ii) $L\left(\dfrac{d^2 f(t)}{dt^2}\right) = s^2F(s) - sf(0^+) - f^{'}(0^+)$

(iii) $L\left(\dfrac{d^3 f(t)}{dt^3}\right) = s^3F(s) - s^2 f(0^+) - sf^1(0^+) - f^{''}(0^+)$

(3) If the Laplace transform of f(t) is F(s) then

(i) $L\left(\int f(t)\right) = \left[\dfrac{F(s)}{s} + \dfrac{f^{-1}(0^+)}{s}\right]$

(ii) $L\left(\iint f(t)\right) = \left[\dfrac{F(s)}{s^2} + \dfrac{f^{-1}(0^+)}{s^2} + \dfrac{f^{-2}(0^+)}{s}\right]$

(iii) $L\left(\iiint f(t)\right) = \dfrac{F(s)}{s^3} + \dfrac{f^{-1}(0^+)}{s^3} + \dfrac{f^{-2}(0^+)}{s^2} + \dfrac{f^{-3}(0^+)}{s}$

(4) If the LT of f(t) is F(s) then $Le^{-at}f(t) = F(s+a)$

(5) if the LT of f(t) is F(s)

$$L\left(tf(t)\right) = -\frac{dF(s)}{ds}$$

(6) Initial value theorem

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} s\,Lf(t) \qquad \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

(7) $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sLf(t) \qquad \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Q.1. Find the invercese LT of the following functions

(i) $F(s) = \dfrac{1}{s(s+1)}$

$$L^{-1}\left(\frac{1}{s(s+1)}\right) = L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{1}{s+1}\right)$$

$$L^{-1}\left(\frac{1}{s(s+1)}\right) = (1-e^{-t})$$

(ii) $\dfrac{s+6}{s(s^2+4s+3)} = \dfrac{s+6}{s(s+1)(s+3)}$

$$L^{-1}\left(F(s)\right) = L^{-1}\left(\frac{2}{s} - \frac{2.5}{s+1} + \frac{\frac{1}{2}}{s+3}\right)$$

$$f(t) = 2 - 2.5e^{-t} + .5e^{-3t}$$

Q.2. Use the Laplace transform to solve the differential equation

$$\frac{d^2v(t)}{dt^2} + 6\frac{dv(t)}{dt} + 8v(t) = 2u(t)$$

$$v(o) = 1, \qquad v'(0) = -2$$

Solution : We take the L.T of each term in given differential equation

$$s^2V(s) - sV(0) - V'(0) + 6\left[sV(s) - V(0)\right] + 8V(s) = \frac{2}{S}$$

$$V(0) = 1, \qquad V'(0) = -2$$

$$s^2V(s) - s + 2 + 6\left[sV(s) - 1\right] + 8V(s) = \frac{2}{s}$$

$$V(s) = \frac{s^2+4s+2}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$A = sV(s)/_{s=0} = \frac{1}{4} \qquad B = (s+2)V(s)/_{s=-2} = \frac{1}{2} \qquad C = (s+4)V(s)/_{s=-4} = \frac{1}{4}$$

$$V(s) = \frac{\frac{1}{4}}{s} + \frac{\frac{1}{2}}{s+2} + \frac{\frac{1}{4}}{s+4}$$

by the Inverse L.T

$$V(t) = \frac{1}{4}(1 + 2e^{-2t} + e^{-4t})u(t)$$

Q.3. Solve for the response y(t) in the following integrodifferential equation

$$\frac{dy}{dt} + 5y(t) + 6\int_0^t y(\tau)d\tau = u(t)$$

$$y(0) = 2$$

Solution : Taking the LT of each term, we get

$$\{SY(s) - Y(0)\} + 5Y(s) + \frac{6}{s}Y(s) = \frac{1}{s}$$

$$y(0) = 2$$

$$Y(s)\{s^2 + 5s + 6\} = 1 + 2s$$

$$Y(s) = \frac{2s+1}{(s+2)(s+3)}$$

$$Y(s) = -\frac{3}{s+2} + \frac{5}{s+3}$$

$$L^{-1}(Y(s)) = (-3e^{-2t} + 5e^{-3t})u(t)$$

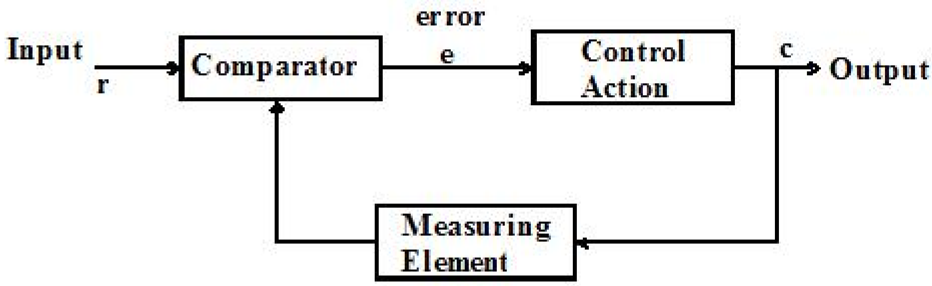
CHAPTER-2

BASICS OF CONTROL SYSTEM

Open–Loop Control System

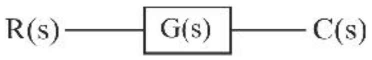
In an open loop Control System the output is neither measured nor feedback for comparison with input. Faithfulness of an open loop control system depends on the accuracy of input calibration.

Closed-Loop Control System ì



In a closed loop control system the output has an effect on control action through a feedback as shown figure and hence closed loop control systems are also termed as feedback control systems. The control action is actuated by an error signal *e* which is the difference b/w the input signal r and the output signal c. This process of comparison b/w the output and input maintains the output at a desired level through control action process.

Open Loop Systems



$$T.F. = \frac{C(s)}{R(s)} = G(s)$$

1. Automatic coffee server
2. Traffic Signal

ADVANTAGES ì

3. Simple and economic

Disadvantages ì

4. Unreliable

Closed Loop Systems

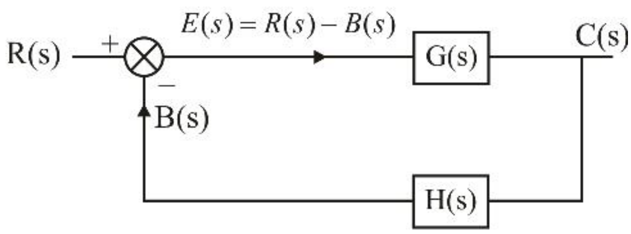


Figure-(a)

1. Electric Iron
2. D.C motor speed control
3. Accurate and reliable
4. The system is complex and costly

5. Inaccurate	5. System may become unstable

From Figure-(a)

$C(s) = G(s).E(s)$ .....(i)

$E(s) = R(s) - B(s)$ .....(ii)

$B(s) = C(s)H(s)$ .....(iii)

From (ii) and (iii) we have

$E(s) = R(s) - C(s).H(s)$ .....(iv)

From (i) and (iv) we have

$$\frac{C(s)}{G(s)} = R(s) - C(s).H(s) \qquad R(s) = C(s)\left[\frac{1+G(s)H(s)}{G(s)}\right] \qquad \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

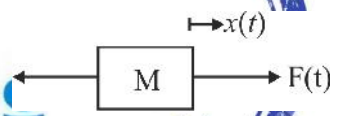
Modelling of a Control System

Translational Motion: Motion takes place along the horizontal or straight line to the section is called translational motion.

FORCES THAT RESIST MOTION

1. **Inertia Force:**

According to Netwon's 2nd law

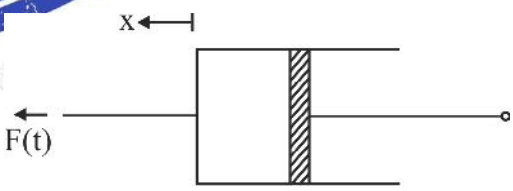


$F = ma$ [In terms of acceleration]

$F = m.\dfrac{dv}{dt}$ [In terms of velocity]

$\boxed{F = m.\dfrac{d^2x}{dt}}$ [In terms of displacement]

2. **Damping Force:**



$F(t) = B.v(t) = B.\dfrac{dx}{dt}$

$\boxed{F(t) = B.\dfrac{dx}{dt}}$

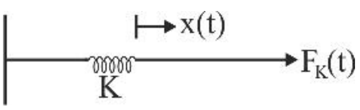
Where B = damping coefficient

unit of B = N/m/sec.

3. **Spring Force:**

The restoring force of a spring is proportional to the displacement.

$F_K(t) = Kx(t)$



$\boxed{F = Kx}$

Unit of K = N/m

Where K = spring constant or stiffness

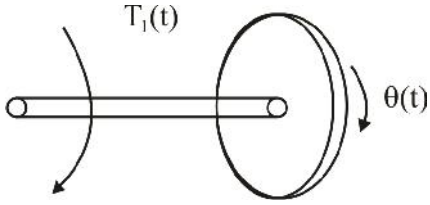
Stiffness = Restoring force per unit displacement.

ROTATIONAL SYSTEM:

The rotational motion of a body can be defined as the motion about a fixed point.

Force that resist motion:

1. **Inertia Torque:**



$$T_1 = J.\frac{d^2 \,\text{–}\,(t)}{dt^2}$$

$\text{–}(t)$ = Angular velocity

J = moment of inertia

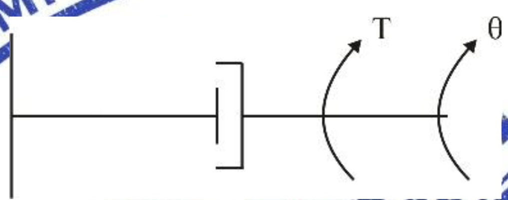
2. **Damping Torque:**

The damping torque $T_D(t)$ is the product of damping coefficient B and angular velocity ω.

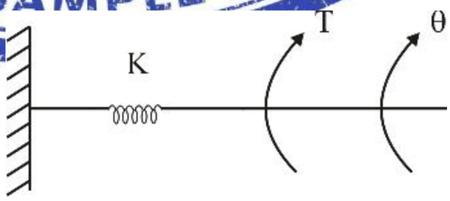
$$T_D(t) = B\mu(t)$$

$$T_D(t) = B.\frac{d}{dt}\text{–}(t)$$

$$T = B.\frac{d\text{–}}{dt}$$



3. **Spring Torque:**



$T = K\text{–}$

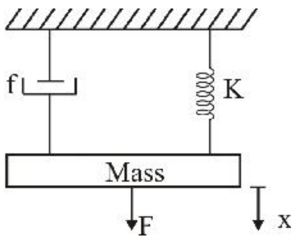
Unit of K = N.m/rad.

D'Alebert's Principle:

The principle states that " for any body the algebraic sum of all forces is equal to zero"

i.e. Applied force + Resisting force = 0

Example 1:



Inertia force $= F_1 = M\frac{d^2x}{dt^2}$

Damping force $= F_2 = f.\frac{dx}{dt}$

Spring force $= F_3 = Kx$

From D'Alembert's principal

$F = F_1 + F_2 + F_3$

$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + kx \qquad ...(1)$$

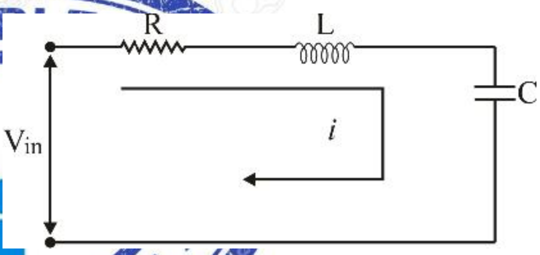
i.e. Applied force = Resisting force.

Procedure of Writing the Equations for Mechanical Systems:

1. Consider the system is in equilibrium
2. Same arbitrary displacement
3. Draw a free body diagram for each mass.
4. Apply Newton's law of motion on each diagram
5. Write the equation in suitable form

Force Voltage Analogy:

➢ Consider the circuit shown below.



➢ Apply K.V.L.

$$v_{in} = iR + L\frac{di}{dt} + \frac{1}{C}\int i.d(t)$$

Since $i = \frac{dq}{dt}$.

$$\therefore \qquad R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{c} = v_{in}$$

On Rearranging

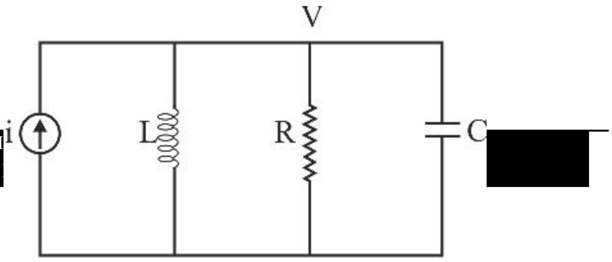
$$V_{in} = L\frac{d^2q}{dt} + \frac{R.dq}{dt} + \frac{q}{c} \qquad ......(2)$$

From example-1 on comparing equation (1) and (2) are can write.

Electrical	Analogous to	Mechanical
L	Analogous to	M
R	Analogous to	f
K	Analogous to	1/C
V_{in}	Analogous to	Force
Q	Analogous to	Displacement 'x'

FORCE CURRENT ANALOGY:

➢ Consider the circuit shown below.



Apply K.C.L.

$$i = \frac{V}{R} + \frac{1}{L}\int v.dt + C.\frac{dV}{dt}$$

Since $v = \frac{d\phi}{dt}$

$$i = \frac{Cd^2\phi}{dt^2} + \frac{1}{R}.\frac{d\phi}{dt} + \frac{\phi}{L}$$

On comparing with the equation (1) of example 1(a) we can write.

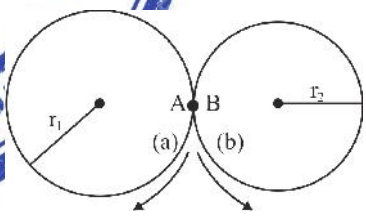
Electrical	Analogous to	Mechanical
C	“	M
1/R	“	f
1/L	“	K
ϕ	“	x
I	“	force

Note: We can represent any mechanical system to a electrical system or vice-versa by the use of modelling.

MECHANICAL COUPLING:

- Consider the two wheels are mechanically coupled.

Consider



$w_1 \,\&\, w_2$ = Angular velocity

$T_1 \,\&\, T_2$ = Torque

$r_1 \,\&\, r_2$ = Radius

$Q_1 \,\&\, Q_2$ = Angular displacement

$N_1 \,\&\, N_2$ = No. of teeth on wheels

- Consider ideal case

  Work done by wheel ‘a’ = work done by wheel ‘b’

$$\therefore \quad T_1Q_1 = T_2Q_2$$

$$\boxed{\frac{T_1}{T_2} = \frac{Q_2}{Q_1}} \qquad \text{……(i)}$$

And linear distance will be

$$\frac{Q_1}{Q_2} = \frac{r_2}{r_1}$$

$$\therefore \quad \boxed{\frac{T_1}{T_2} = \frac{Q_2}{Q_1} = \frac{r_1}{r_2}} \quad .....(ii)$$

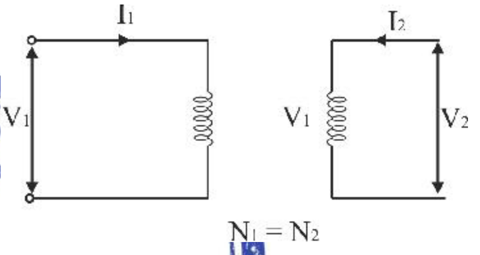
Since the n.o. of teeth is proportional to radius. Thus

$$\frac{N_1}{N_2} = \frac{r_1}{r_2}$$

And $\frac{w_1}{w_2} = \frac{Q_2}{Q_1}$

So $$\boxed{\frac{T_1}{T_2} = \frac{Q_2}{Q_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{w_2}{w_1}} \quad .....(iii)$$

From force-voltage analogy



Where

$V_1 \rightarrow$ is analogous to $\rightarrow T_1$

$V_2 \rightarrow$ is analogous to $\rightarrow T_2$

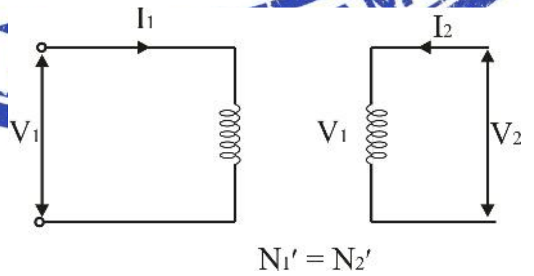
$I_1 \rightarrow$ is analogous to $\rightarrow w_1$

$I_2 \rightarrow$ is analogous to $\rightarrow w_2$

$r_1 \rightarrow$ is analogous to $\rightarrow N_1$

$r_2 \rightarrow$ is analogous to $\rightarrow N_2$

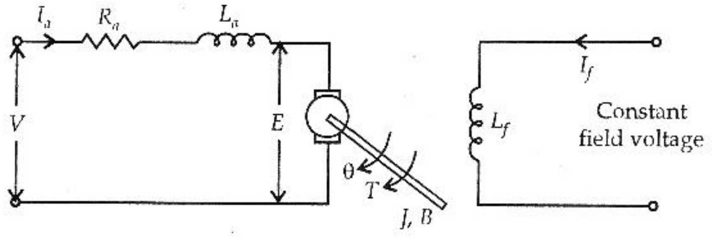
- From force-current analogy



Question: Show the electrical connection diagram and model the armature controlled d.c. motor **in** a block diagram form. Assume the necessary variables and obtained transfer function for change in position of armature to the change in armature voltage. Express the transfer function in standard form.

IES-2011-EE

Solutions: Armature Controlled d.c. Motor



Figure

Consider the armature controlled d.c. motor and assume that the demagnetizing effect of armature reaction is neglected, magnetic circuit is assumed linear and field voltage is constant i.e. I_f = constant

Let $R_a =$ Armature resistance

$L_a =$ **Armature self inductance caused by armature flux**

i_f = **field current**

E = **Induced e.m.f in armature**

V = **Applied voltage**

T = **Torque developed by the motor**

θ = **Angular displacement of the motor shaft**

J = **Equivalent moment of inertia of motor shaft & load referred to the motor**

B = **equivalent coefficient of friction of motor and load referred to the motor**

Apply KVL in armature circuit

$$V = R_a i_a + L\frac{di_a}{dt} + E \qquad \text{.....(1)}$$

Since, field current I_f Constant, the flux ¤ will be constant

When armature is rotating, an e.m.f is induced

$$E \sim ¤\,\mu$$

$$E = K_b \mu$$

or ,

$$E = K_b \frac{d¯}{dt} \qquad \text{.....(2)}$$

Where μ = angular velocity

$K_b =$ Back e.m.f constant

Now, the torque T delivered by the motor will be the product of armature current and flux

$$T \sim ¤\, i_a$$

$$T = K\, i_a \qquad \text{.....(3)}$$

Where K = motor torque constant

The dynamic equation with moment of inertia & coefficient of friction will be

$$T = J\frac{d^2¯}{dt^2} + B\frac{d¯}{dt} \qquad \text{.....(4)}$$

Take the Laplace transform of equations 1, 2, 3 and 4

$$V(s) - E(s) = I_a(s)(R_a + SL_a)$$

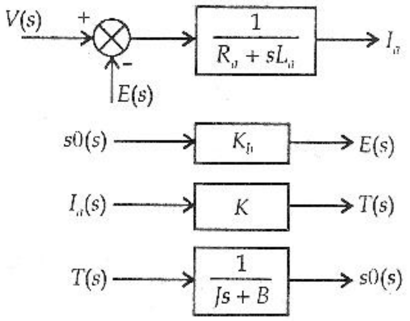
$$E(s) = K_b S¯(s)$$

$$T(s) = K\,I_a(s)$$

$$T(s) = (s^2 J + sB)^{-}(s)$$

$$T(s) = (SJ + B)S^{-}(s)$$

The block diagram for each equation



Combine all four block diagrams

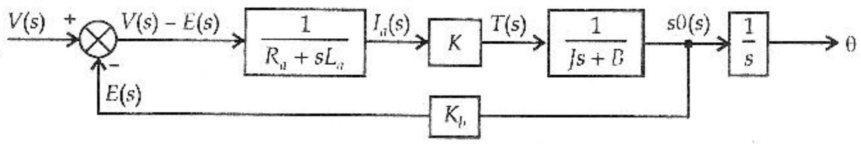
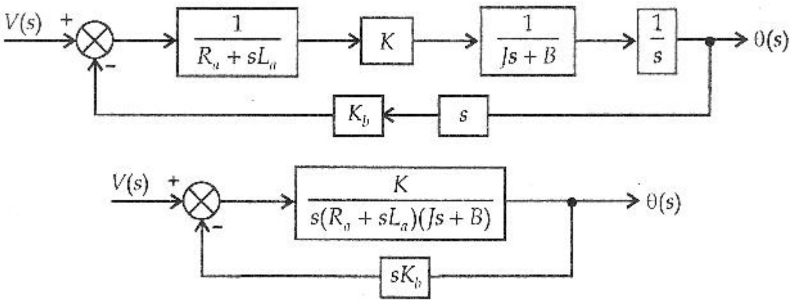


Figure : Block diagram of armature controlled d.c. motor

Now determine the transfer function by block reduction method.



$$\frac{^{-}(s)}{V(s)} = \frac{K}{(R_a + sL_a)(Js + B)s + KK_b s} \qquad \text{....(5)}$$

Equation 5 can be written as

$$\frac{^{-}(s)}{V(s)} = \frac{K}{R_a\left(1 + s\dfrac{L_a}{R_a}\right)sB\left(1 + s.\dfrac{J}{B}\right) + KK_b s}$$

Put $\dfrac{L_a}{R_a} = 2_a$ **time constant of armature circuit**

$\dfrac{J}{B} = 2_m =$ **Mechanical time constant**

Equation 5 becomes

$$\frac{\overline{\ }(s)}{V(s)} = \frac{K}{sR_a B(1+s^2{}_a)(1+s^2{}_m)+KK_b s} \qquad \text{.....(6)}$$

From the block diag. figure it is clear that it is a closed loop system. The effect of the back e.m.f is represented by the feedback signal proportional to the speed of the motor.