SAMPLE STUDY MATERIAL

Electronics Engineering

EC / E & T

Postal Correspondence Course

GATE , IES & PSUs

Signal System

CONTENTS

CHAPTER-1

SIGNALS AND SYSTEM

1.1 INTRODUCTION

The concept and theory of signals and systems are needed in almost all electrical engineering fields and in many other engineering and scientific disciplines as well. In this chapter we introduce the mathematical description and representation of signals and systems and their classifications. We also define several important basic signals essential to our studies.

What are Signals?

Signals are represented as functions of one or more independent variables. For example, a digital image can be represented by intensity as a function of time. All signals carry some kind of information.

Classification of signals

Signals can be classified on the basis of different parameters:

A. Continuous time and discrete time signals

A signal is said to be continuous time signal if it is defined at all values of time parameter '*t*' or we can say '*t*' must be continuous variable.

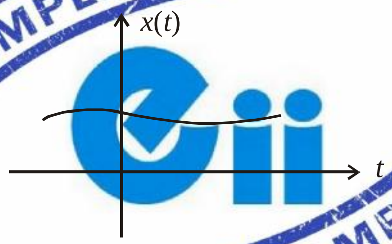


Fig. 1 (a)

A discrete-time signal can be obtained from sampling a continuous time signal $x(t)$ at discrete time instants like t = 0, 1, 2, ……. It is represented by $x[n]$. Various values are given by $x[0]$, $x[1]$ …….

Fig. 1 (b)

$x[0]$, $x[1]$ …… are called samples of the continuous time signal $x(t)$.

For example: $$x[n]=\begin{cases}\left(\dfrac{1}{3}\right)^n & n\geq 0\\ 0 & n<0\end{cases}\qquad n = \ldots\ldots\ -2, -1, 0, 1, 2, \ldots\ldots$$

B. Analog and Digital Signals

If a continuous-time signal $x(t)$ can take on any value in the continuous interval (a, b), where a may be $-\infty$ and b may be $+\infty$, then the continuous-time signal $x(t)$ is called an analog signal. If a discrete-time signal $x[n]$ can take on only a finite number of distinct values, then we call this signal a digital signal. Digital signals are discretized in both time and value.

C. Real and Complex Signals

A signal $x(t)$ is a real signal if its value is a real number, and a signal $x(t)$ is a complex signal if its value is a complex number. A general complex signal $x(t)$ is a function of the form

$$x(t) = x_1(t) + jx_2(t) \qquad \text{.... (1.1)}$$

Where $x_1(t)$ and $x_2(t)$ are real signal and $j = \sqrt{-1}$.

Note that in Eq. (1.1) t represent either a continuous or a discrete variable.

D. Deterministic and Random Signals

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time t. Random signals are those signals that take random values at any given time and must be characterized statistically. Random signals will not be discussed in this text.

For example: $y(t) = 2t + 1$

So $y(t)$ is defined and deterministic for each value of t.

And suppose temperature of a city is a random signal which can take arbitrary values.

E. Even and Odd Signals

A signal $x(t)$ or $x[n]$ is referred to as an even signal if

$$x(-t) = x(t)$$
$$x[-n] = x[n] \qquad \text{....(1.2)}$$

A signal $x(t)$ or $x[n]$ is referred to as an odd signal if

$$x(-t) = -x(t)$$
$$x[-n] = -x[n] \qquad \text{...(1.3)}$$

Examples of even and odd signals are shown in Fig. 1.2.

Any signal $x(t)$ or $x[n]$ can be expressed as a sum of two signals, one of which is even and one of which is odd. That is,

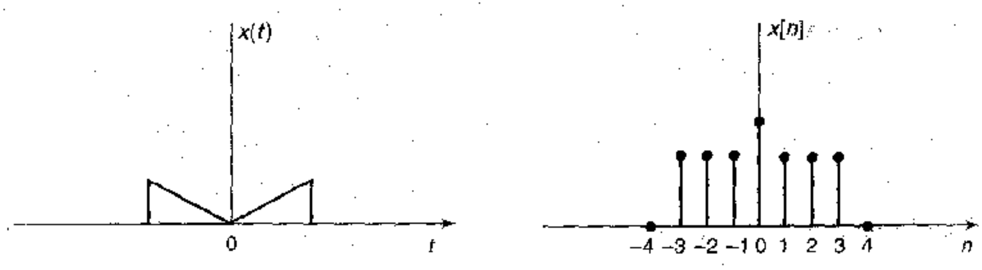
$$x(t) = x_e(t) + x_o(t) \qquad \text{...(1.4)}$$
$$x[n] = x_e[n] + x_o[n]$$

where $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$ even part of $x(t)$

$$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\} \text{ even part of } x[n] \qquad \text{...(1.5)}$$

$$x_0(t) = \frac{1}{2}\{x(t) - x(-t)\} \text{ odd part of } x(t)$$

$$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\} \text{ odd part of } x[n] \qquad \text{...(1.6)}$$



(a) (b)

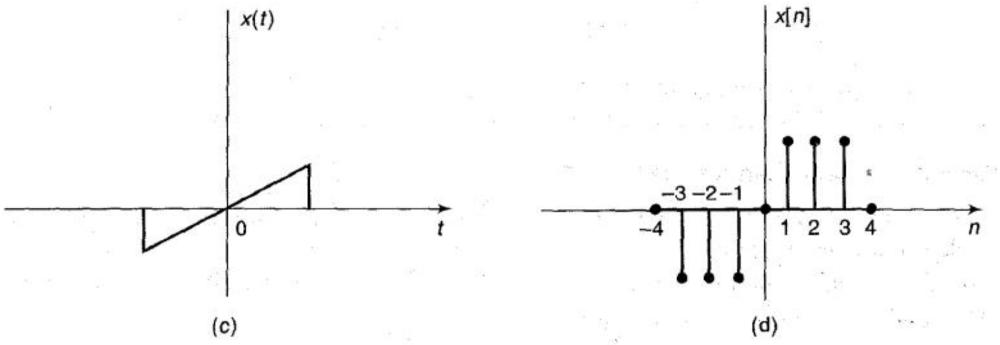


Fig. 1.2 *Examples of Even Signals (a and b) and Odd Signals (c and d)*

So, from the above formulae, we can split any signal into its even and odd parts. And subsequently we can find the even and odd part of any signal $x(t)$.

Note that the product of two even signals or of two odd signals is an even signal and that the product of an even signal and an odd signal is an odd signal (Solved Problem 1.7).

F. Periodic and Non-periodic Signals

A continuous-time signal $x(t)$ is said to be periodic with period T if there is a positive nonzero value of T for which

$$x(t) = x(t+T) \quad \text{for all } t \qquad \ldots(1.7)$$

An example of such a signal is given in Fig. 1.3(a). From Eq. (1.7) or Fig. 1.3(a) it follows that

$$x(t+mT) = x(t) \qquad \ldots(1.8)$$

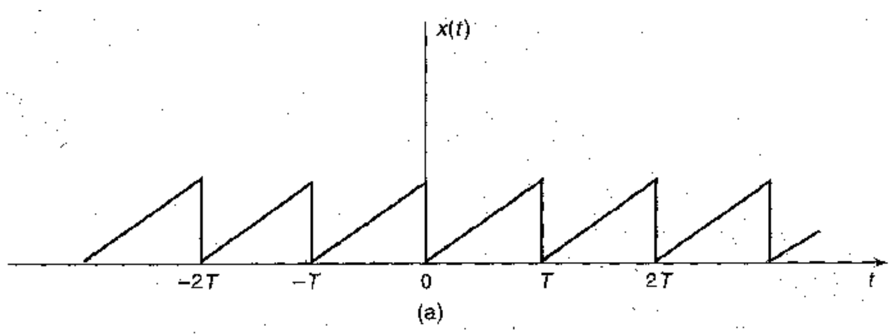
For all t and any integer m. The fundamental period T_0 of $x(t)$ is the smallest positive value of T for which Eq. (1.7) holds. Any continuous-time signal which is not periodic is called a non-periodic (or aperiodic).

Periodic discrete-time signals are defined analogously. A sequence (discrete-time signal) $x[n]$ is periodic with period N if there is a positive integer N for which

$$x[n+N] = x[n] \quad \text{all } n \qquad \ldots(1.9)$$

An example of such a sequence is given in Fig. 1.3(b). From Eq. (1.9) and Fig. 1.3(b) it follows that

$$x[n+mN] = x[n] \qquad \ldots(1.10)$$



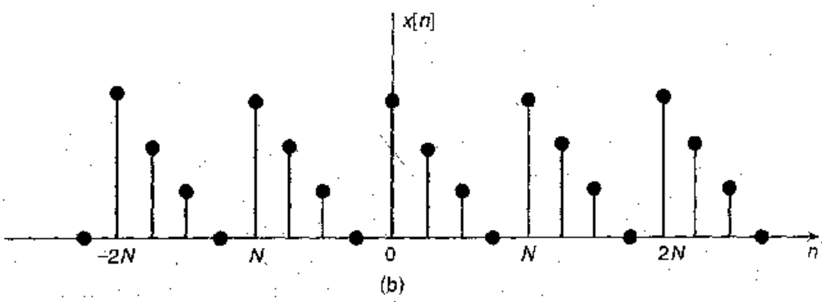


Fig. 1.3 *Examples of Periodic Signals*

for all n and any integer m. The fundamental period N_0 of $x[n]$ is the smallest positive integer N for which Eq. (1.9) holds. Any sequence which is not periodic is called a non-periodic (or aperiodic) sequence.

Note: (i) Fundamental period of constant signal is undefined.

(ii) Sum of two continuous periodic signals may not be periodic but sum of two periodic sequences is always periodic.

G. Energy and Power Signals

For an arbitrary continuous-time signal $x(t)$, the normalized energy content E of $x(t)$ is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad \ldots (1.11)$$

The normalized average power P of $x(t)$ is defined as

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \qquad \ldots (1.12)$$

Similarly, for a discrete-time signal $x[n]$, the normalized energy content E of $x[n]$ is defined as

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad \ldots (1.13)$$

The normalized average power P of x[n] is defined as

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \qquad \ldots (1.14)$$

For Example: Consider $v(t)$ to be the voltage across a resistor R producing a current $i(t)$. The instantaneous power $p(t)$ per ohm in defined as

$$p(t) = \frac{v(t)\, i(t)}{\text{R}} = i^2(t) \qquad \ldots (1.15)$$

Total energy E and average power P on a per ohm basis are

$$\text{E} = \int_{-\infty}^{\infty} i^2(t)\, dt. \text{ joules} \qquad \ldots (1.16)$$

$$\text{And P} = \lim_{\text{T}\to\infty} \frac{1}{\text{T}} \int_{-\frac{\text{T}}{2}}^{\frac{\text{T}}{2}} i^2(t)\, dt \text{ watts} \qquad \ldots (1.17)$$

Based on definitions (1.11) to (1.14), the following classes of signals are defined:

1. $x(t)$ (or $x[n]$) is said to be an energy signal (or sequence) if and only if $0 < E < \infty$, and so $P = 0$.
2. $x(t)$ (or $x[n]$) is said to be a power signal (or sequence) if and only if $0 < P < \infty$, thus implying that E

$= \infty$.

3. Signals that satisfy neither property are referred to as neither energy signals nor power signals.

Note that a periodic signal is a power signal if its energy content per period is finite, and then the average power of this signal need only be calculated over a period.

1.3 BASIC CONTINUOUS-TIME SIGNALS

A. The unit step functions

The unit step function u(t), also known as the Heaviside unit function, is defined as

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Which is shown in Fig. 1.4(a) Note that it is discontinuous at t = 0 and that the value at t = 0 is undefined. Similarly, the shifted unit step function $u(t - t_0)$ is defined as

$$u(t - t_0) = \begin{cases} 1 & t > t_0 \\ 0 & t < t_0 \end{cases}$$

This is shown in fig 1.4(b)

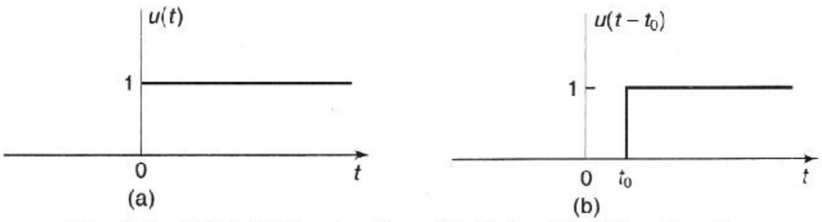


Fig. 1.4 (a) Unit Step Function; (b) Shifted Unit Step Function

B. The unit impulse function

The unit impulse function δ(*t*), also known as the Dirac delta function, plays a central role in system analysis. Traditionally, δ(*t*) is often defined as the limit of a suitably chosen conventional function having unity area over an infinitesimal time interval as shown in Fig. 1.5 and possesses the following properties:

$$\text{¢}(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

$$\int_{-\text{£}}^{\text{£}} \text{¢}(t)\,dt = 1$$

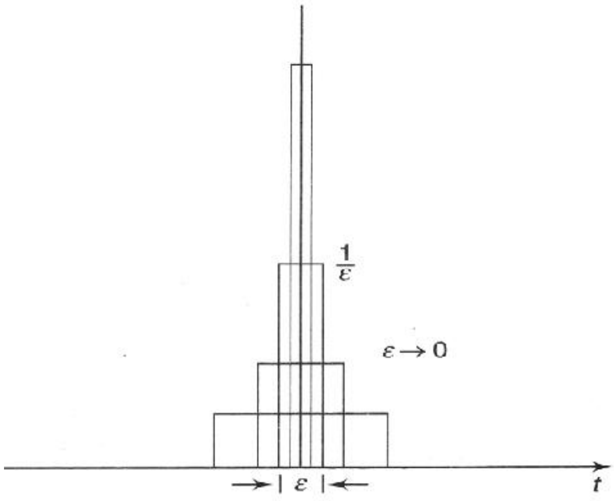


Fig. 1.5

But an ordinary function which is everywhere 0 except at a single point must have the integral 0 (in the Riemann integral sense). Thus δ(t) cannot be an ordinary function and mathematically it is defined by

$$\int_{-\infty}^{\infty}\phi(t)\delta(t)\,dt=\phi(0) \qquad \dots(1.18)$$

Where $\phi(t)$ is any regular function continuous at t = 0.

An alternative definition of δ(t) is given by

$$\int_a^b \phi(t)\delta(t)\,dt=\begin{cases}\phi(0) & a<0<b\\ 0 & a<b<0 \;\; or \;\; 0<a<b\\ undefined & a=0 \;\; or \;\; b=0\end{cases} \qquad \dots(1.19)$$

Note that equation (1.18) to (1.19) is a symbolic expression and should not be considered an ordinary Riemann integral. In this sense, δ(t) is often called a generalized function and ϕ(t) is known as a testing function.

For convenience, $\delta(t)$ and $\delta(t-t_0)$ are depicted graphically as shown in Fig. 1.6.

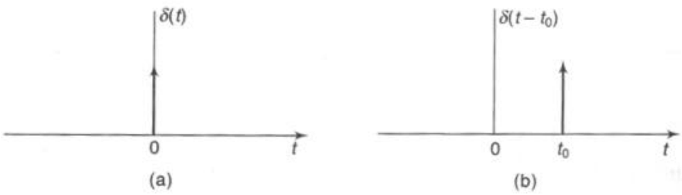


Fig. 1.6 (a) Unit Impulse Function; (b) Shifted Unit Impulse Function

Some additional properties of δ(t) are

$$\delta(at)=\frac{1}{|a|}\delta(t) \qquad \dots (1.20)$$

$$\delta(-t)=\delta(t) \qquad \dots (1.21)$$

$$x(t)¢(t) = x(0)¢(t) \qquad \dots (1.22)$$

If $x(t)$ is continuous at t = 0.

$$x(t)¢(t-t_0) = x(t_0)¢(t-t_0) \qquad \dots (1.23)$$

If $x(t)$ is continuous at $t = t_0$.

Using Equations (1.19) and (1.21), any continuous-time signal $x(t)$ can be expressed as

$$x(t) = \int_{-\infty}^{\infty} x(²)¢(t-²)\,d² \qquad \dots (1.24)$$

Note: Unit step functions and unit impulse function are very much used in networks, control systems. So better understanding is necessary for these signals.

C. Complex Exponential Signals:

The complex exponential signal

$$x(t) = e^{jµ_0 t} \qquad \dots (1.25)$$

is an important example of a complex signal. Using Euler's formula, this signal can be defined as

$$x(t) = e^{jµ_0 t} = \cos µ_0 t + j \sin µ_0 t \qquad \dots (1.26)$$

Thus, $x(t)$ is a complex signal whose real part is $\cos µ_0 t$ and imaginary part is $\sin µ_0 t$. An important property of the complex exponential signal $x(t)$ in Eq. (1.25) is that it is periodic. The fundamental period T_0 of $x(t)$ is given by (Solved Problem 1.9)

$$T_0 = \frac{2®}{|µ_0|} \qquad \dots (1.27)$$

Note that $x(t)$ is periodic for any value of $µ_0$

General Complex Exponential Signals Let $s = ± + jµ$ be a complex number. We define $x(t)$ as

$$x(t) = e^{st} = e^{(± + jµ)t} = e^{±t}(\cos µt + j \sin µt) \qquad \dots (1.28)$$

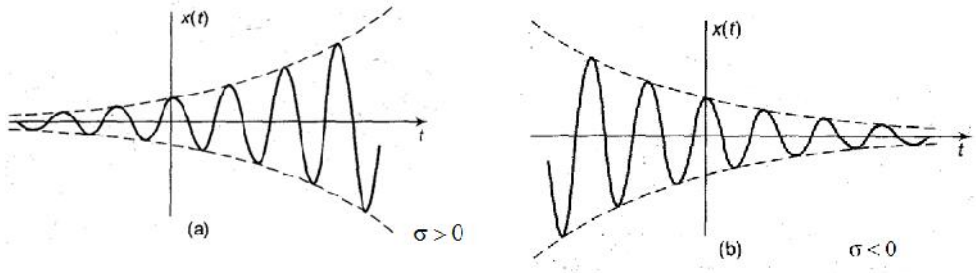


Figure-1.7: (a) Exponentially Increasing sinusoidal signal (b) Exponentially decreasing sinusoidal Signal

Then signal $x(t)$ in Eq. (1.27) is known as a general complex exponential signal whose real part $e^{\pm t}\cos \mu t$ and imaginary part $e^{\pm t}\sin \mu t$ are exponentially increasing $(\pm > 0)$ or decreasing $(\pm < 0)$ sinusoidal signals (Fig 1.7).

Real exponential signals Note that if $s = \pm$ (a real number), then Eq. (1.28) reduces to a real exponential signal

$$x(t) = e^{\pm t} \quad \text{… (1.29)}$$

As illustrated in Figure below, if $\pm > 0$, then $x(t)$ is a growing exponential; and if $\pm < 0$, then $x(t)$ is a decaying exponential.

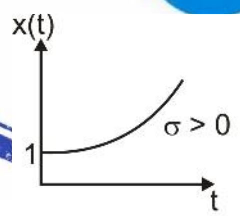


Fig: 8

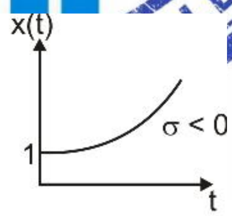


Fig: 9

1.4 BASIC DISCRETE-TIME SIGNALS

A. The Unit Step Sequence

The unit step sequence $u[n]$ is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \quad \text{… (1.30)}$$

Which is shown in Fig. 1.10(a). Note that the value of $u[n]$ at $n = 0$ is defined [unlike the continuous-time step function $u(t)$ at t = 0] and equals unity. Similarly, the shifted unit step sequence $u[n - k]$ is defined as

$$u[n-k] = \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases} \quad \text{… (1.31)}$$

Which is shown in Fig. 1.10(b).

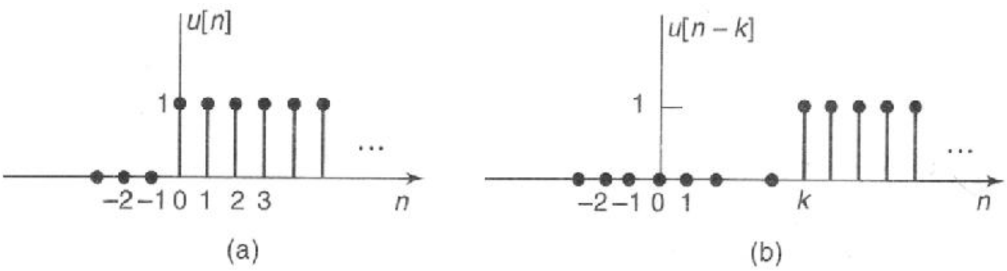


Fig. 1.10 (a) Unit Step Sequence; (b) Shifted Unit Step Sequence

B. The Unit Impulse Sequence

The unit impulse (or unit sample) sequence δ[n] is defined as

$$\delta[n]=\begin{cases}1 & n=0\\ 0 & n\neq 0\end{cases} \quad \ldots (1.32)$$

Which is shown in Fig. 1.11(a). Similarly, the shifted unit impulse (or sample) sequence $\delta[n-k]$ is defined as

$$\delta[n-k]=\begin{cases}1 & n=k\\ 0 & n\neq k\end{cases} \quad \ldots (1.33)$$

Which is shown in Fig. 1.11(b).

Unlike the continuous-time unit impulse function $\delta(t), \delta[n]$ is defined without mathematical complication or difficulty. From definitions (1.32) and (1.33) it is readily seen that

$$x[n]\delta[n]=x[0]\delta[n] \quad \ldots (1.34)$$

$$x[n]\delta[n-k]=x[k]\delta[n-k] \quad \ldots (1.35)$$

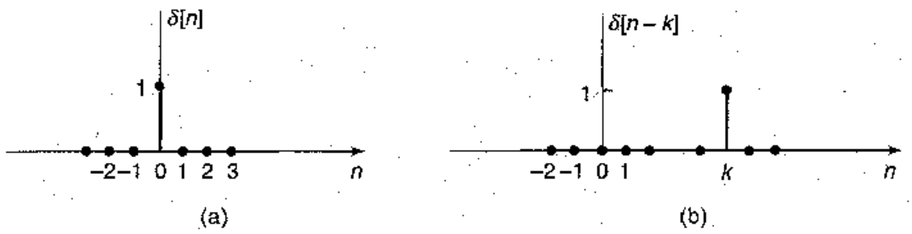


Fig. 1.11 (a) Unit Impulse (Sample) Sequence; (b) Shifted Unit Impulse Sequence

Which are the discrete-time counterparts of Equations (1.21) and (1.22), respectively. Above property is called sifting property. From definitions (1.30) to (1.31), δ [n] and $u[n]$ are related by

$$\delta[n] = u[n] - u[n-1] \quad \ldots (1.36)$$

$$u[n]=\sum_{k=-\infty}^{n}\delta[k] \quad \ldots (1.37)$$

Which are the discrete-time counterparts of Equations (1.30) and (1.31), respectively.

Using definition (1.33), any sequence $x[n]$ can be expressed as

$$x[n] = \sum_{k=-\infty}^{n} x[k]\delta[n-k] \qquad \ldots (1.38)$$

Which corresponds to Eq. (1.27) in the continuous-time signal case.

C. Complex Exponential Sequences

The complex exponential sequence is of the form

$$x[n] = e^{j\Omega_0 n} \qquad \ldots (1.39)$$

Again, using Euler's formula, $x[n]$ can be expressed as

$$x[n] = e^{j\Omega_0 n} = \cos\Omega_0 n + j\sin\Omega_0 n \quad \ldots (1.40)$$

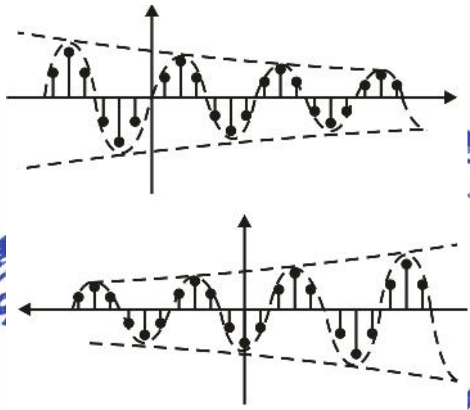


Fig: 1.12

Thus $x[n]$ is a complex sequence whose real part is $\cos\Omega_0 n$ and imaginary part is $\sin\Omega_0 n$.

Periodicity of $e^{j\Omega_0 n}$ is order for $e^{j\Omega_0 n}$ to be periodic with period N (> 0), $\Omega_0 n$ must satisfy the following condition,

$$\frac{\Omega_0}{2\pi} = \frac{m}{N} \qquad m = \text{positive integer} \quad \ldots (1.41)$$

Thus, the sequence $e^{j\Omega_0 n}$ is not periodic for any value of Ω_0. It is periodic only if $\Omega_0 / 2\pi$ is a rational number. Note that this property is quite different from the property that the continuous- time signal $e^{j\omega_0 t}$ is periodic for any value of $\cos\omega_0$. Thus, if Ω_0 satisfies the periodicity condition in Eq. (1.41), $\Omega_0 \neq 0$, N and m have no factors in common, then the fundamental period of the sequence $x[n]$ in Eq. (1.39) is N_0 given by

$$N_0 = m\left(\frac{2\pi}{\Omega_0}\right) \qquad \ldots (1.42)$$

Another very important distinction between the discrete-time and continuous-time complex exponentials is that the signals $e^{j\mu_0 t}$ are all distinct for distinct values of μ_0 but that this is not the case for the signals $e^{j\Omega_0 n}$.

Consider the complex exponential sequence with frequency $(\Omega_0 + 2®k)$, where *k* is an integer:

$$e^{j(\Omega_0+2®k)n} = e^{j\Omega_0 n} e^{j2®kn} = e^{j\Omega_0 n} \qquad \text{… (1.43)}$$

Since $e^{j2®kn} = 1$, From Eq. (1.56) we see that the complex exponential sequence at frequency Ω_0 is the same as that at frequencies $(\Omega_0 \pm 2®), (\Omega_0 \pm 4®)$, and so on. Therefore, in dealing with discrete- time exponentials, we only consider an interval of length 2® and usually, we will use the interval 0 $0 \le \Omega_0 < 2®$ or the interval $-® \le \Omega_0 < ®$.

So in frequency domain, these complex exponential sequences are periodic with period 2π.

D. Sinusoidal Sequences

A sinusoidal sequence can be expressed as

$$x[n] = A\cos(\Omega_0 n + ¯) \qquad \text{… (1.44)}$$

If *n* is dimensionless, then both Ω_0 and θ have units of radians. Two examples of sinusoidal sequences are shown in Fig. 1.13. As before, the sinusoidal sequence in Eq. (1.44) can be expressed as

$$A\cos(\Omega_0 n + ¯) = A\,\mathrm{Re}\left\{e^{j(\Omega_0 n + ¯)}\right\} \qquad \text{… (1.45)}$$

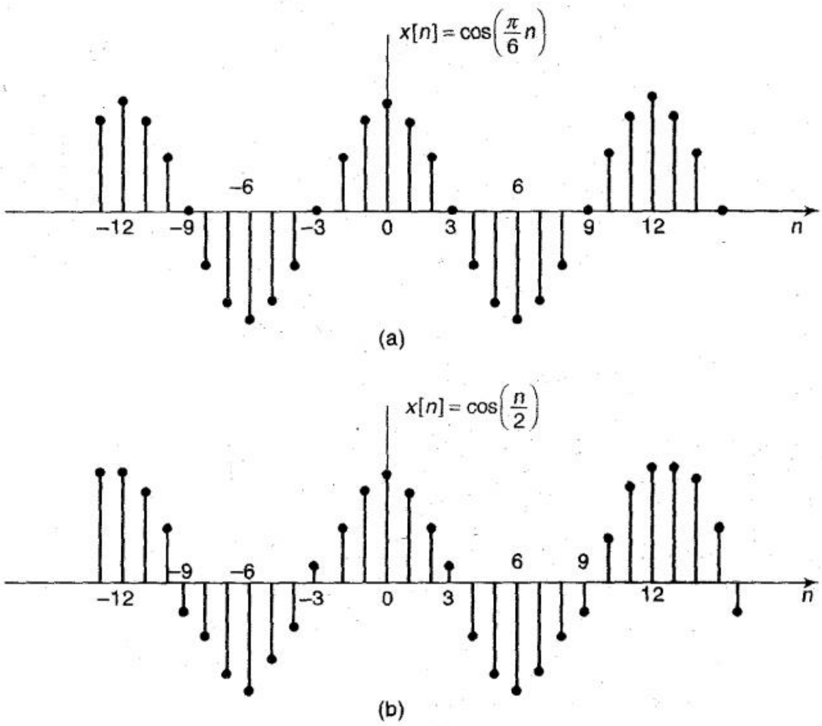


Fig. 1.13 *Sinusoidal Sequences. (a) $x[n] = \cos(\pi n/6)$; (b) $x[n] = \cos(n/2)$*

As we observed in the case of the complex exponential sequence in Eq. (1.39), the same observations [Equations (1.41) and (1.43)] also hold for sinusoidal sequences. For instance, the sequence in Fig. 1.13(a) is periodic with fundamental period 12, but the sequence in Fig. 1.13(b) is not periodic.

1.5 SYSTEMS AND CLASSIFICATION OF SYSTEMS

A. System Representation

A system is a mathematical model of a physical process that relates the input (or excitation) signal to the output (or response) signal.

Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of x into y. This transformation is represented by the mathematical notation

$$y = \mathbf{T}\{x\} \qquad \ldots (1.46)$$

Where **T** is the operator representing some well-defined rule by which x is transformed into y. Relationship (1.46) is depicted as shown in Fig. 1.14(a). Multiple input and/or output signals are possible as shown in Fig. 1.14(b). We will restrict our attention for the most part in this text to the single-input, single-output case.

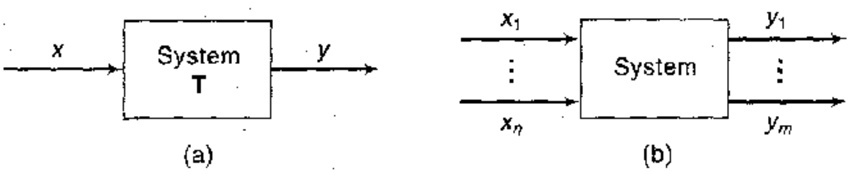


Fig. 1.14 *System with Single or Multiple Input and Output Signals*

B. Continuous-Time and Discrete-Time Systems

If the input and output signals x and y are continuous-time signals, then the system is called a system [Fig. 1.15(a)]. If the input and output signals are discrete-time signals or sequences, then the system is called a discrete-time system [Fig. 1.15(b)].

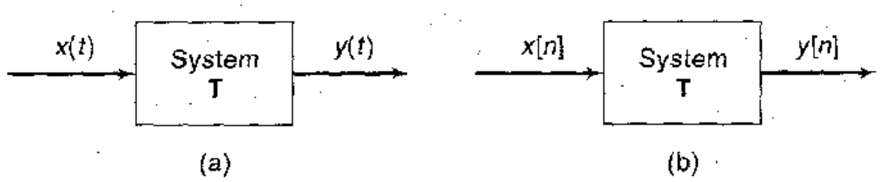


Fig. 1.15 *(a) Continuous-time System; (b) Discrete-time System*

For Example: $y(t) = 2x(t)$ is an example of continuous time system.

And $y(n) = x(n - k)$ is an example of discrete time system.