

Digital Logic

Computer Science & Information Technology (CS)



RANK 1 GATE 2015

Computer Science
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20 Rank under AIR 100

Postal Correspondence

- ✓ Examination Oriented Theory, Practice Set
- ✓ Key concepts, Analysis & Summary



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GATE Syllabus

Digital Logic: Logic functions, Minimization, Design and synthesis of combinational and sequential circuits; Number representation and computer arithmetic (fixed and floating point).

CHAPTER-1

BINARY SYSTEM

Base Conversion: A number $a_n, a_{n-1} \dots a_2, a_1 a_0 \cdot a_{-1} a_{-2} a_{-3} \dots$ expressed in a base r system has coefficient multiplied by powers of r .

$$a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + \dots + a_1 r + a_0 + a_{-1} r^{-1} + a_{-2} r^{-2} + a_{-3} r^{-3} + \dots \quad \dots(A)$$

Coefficients a_j ; range from 0 to $r - 1$

Key Points:

To convert a number of base r to decimal is done by expanding the number in a power series as in (A)

Then add all the terms.

Example 1: Convert following Binary number $(11010.11)_2$ in to decimal number.

Solution:

Base $r = 2$

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$(11010.11)_2 = (26.75)_{10}$$

Example 2: Convert $(4021.2)_5$ in to decimal equivalent

$$\text{Solution: } 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.4)_{10}$$

Example 3: Convert $(127.4)_8$ in to decimal equivalent.

$$\text{Solution: } 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$$

Numbers with Different bases:

Decimal (r = 10)	Binary (r = 2)	Octal (r = 8)	Hexadecimal (r = 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8

09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Example 4: Convert following hexadecimal number into decimal number: $(B65F)_{16}$

Solution: $11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46.687)_{10}$

Conversion of decimal number to a number in base r:

- Separate the number into an integer part and fraction part.
- Divide the number and all successive quotients by r and accumulating the remainders.
- Conversion of decimal fraction is done by multiplying the fraction and all successive fraction and integers are accumulated.

Example 1: Convert decimal number 41 to binary.

Solution:

	Integer quotient	Remainder	Coefficient	
41/2 =	20	1	1	$a_0 = 1$
20/2 =	10	0	0	$a_1 = 0$
10/2 =	5	0	0	$a_2 = 0$
5/2 =	2	1	1	$a_3 = 1$
2/2 =	1	0	0	$a_4 = 0$
1/2 =	0	1	1	$a_5 = 1$

$(101001)_2$

$$(41)_{10} \rightarrow (101001)_2$$

Example 2: Convert $(153)_{10}$ to octal.

Solution:

Required base r is 8.

153 are divided by 8 to give integer quotient of 19 and remainder 1. Then 19 are divided by 8 to give integer quotient of 2 and remainder 3. Finally 2 are divided by 8 to give quotient of 0 and remainder of 2.

$$\begin{array}{r|l} 153 & \\ 19 & 1 \\ 2 & 3 \\ 0 & 2 \end{array} \quad \begin{array}{l} \uparrow \\ \downarrow \\ \leftarrow \end{array} \quad (231)_8$$

Thus $(153)_{10} \rightarrow (231)_8$

Example 3: Convert $(0.6875)_{10}$ to Binary.

Solution: 0.6875 is multiplied by 2 to give an integer and a fraction. The new fraction is multiplied by 2 to give a new integer and new fraction.

This process is continuing until the fraction becomes zero or until the numbers of digits have sufficient accuracy.

	Integer		Fraction		Coefficient
0.6875×2	=	1	+	0.3750	$a_{-1} = 1$
0.3750×2	=	0	+	0.7500	$a_{-2} = 0$
0.7500×2	=	1	+	0.5000	$a_{-3} = 1$
0.500×2	=	1	+	0.0000	$a_{-4} = 1$

$$(0.6875)_{10} \rightarrow (0.1011)_2$$

Example 4: Convert $(0.513)_{10}$ to octal.

Solution:

0.513×8	=	4	+	0.104	$a_{-1} = 4$
0.104×8	=	0	+	0.832	$a_{-2} = 0$
0.832×8	=	6	+	0.656	$a_{-3} = 6$
0.656×8	=	5	+	0.248	$a_{-4} = 5$
0.248×8	=	1	+	0.984	$a_{-5} = 1$
0.984×8	=	7	+	0.872	

Answer to seven significant figures is:

$$(0.406517\dots)_8$$

Thus $(0.513)_{10} \rightarrow (0.406517)_8$

$$(41.6875)_{10} \rightarrow (101001.1011)_2$$

$$(153.513)_{10} \rightarrow (231.406517)_8$$

Octal and hexadecimal numbers:

Conversion from binary to octal is easily done by partitioning the binary number into groups of 3 digits each starting from binary point & proceeding to left and to the right.

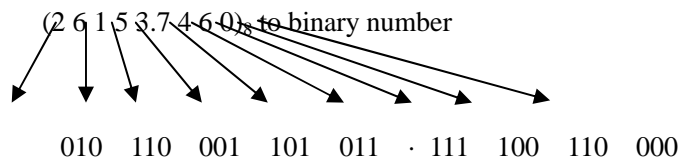
The corresponding octal digit is then assigned to each group.

For conversion into hexadecimal, binary number is divided into group of 4 digits.

Example:

$(26153.7460)_8$ to binary number

Solution:



Thus binary number is

$$(010\ 110\ 001\ 101\ 011.11110011000)_2$$

Example 5: Convert binary to hexadecimal number:

$$(10\ 1100\ 0110\ 1011.1111\ 0010)_2$$

$$0010\ 1100\ 0110\ 1011.1111\ 0010$$

$$\begin{array}{cccccc} 2 & C & 6 & B & F & 2 \\ \hline & & & & & \end{array} \quad 2 = (2C6B.F2)_{16}$$

Example 6: $(673.124)_8$ to binary number:

$$(673.124)_8 \equiv (110\ 111\ 011 \cdot 001\ 010\ 100)_2$$

6 7 3 1 2 4

$(306.D)_{16}$ to binary number:

$$(306.D)_{16} \equiv (0011\ 0000\ 0110 \cdot 1101)_2$$

3 0 6 D

Note: In communication, octal or hexadecimal represented is more desirable because it can be expressed more compactly with a third or a quarter of the number of digits required for the equivalent binary number.

Complements: Complements are used in digital computer for simplifying the subtraction operations and for logic manipulation.

There are 2 types of complements for each base r system

1. Radix complements (r 's complement)
2. Diminished radix complement ($(r - 1)$'s complement)
 1. Diminished radix complement:
 - Given a number N in base r having n digits, the $(r - 1)$'s complement of N is defined as $(r^n - 1) - N$.
 - For decimal number $r = 10$, $(r - 1)$'s complement or 9's complement of N is $(10^n - 1) - N$.

9's complement: $(10^n - 1) - N$

- 10^n can be represented as single 1 followed by n 0's
- $10^n - 1$ is number represented by n 9's.

- Thus 9's complement can be obtained by subtracting each digit of number N by 9's.

Example 7: Find 9's complement of 546700

Solution:

$$999999 - 546700 = 453299$$

9's complement of 546700 is 453299

1's Complement for binary number:

- It is given as $(2^n - 1) - N$
- 2^n can be representing as binary number consist of single 1 followed by n 0's.
- $2^n - 1$ can be represented as n 1's.

Example 8: $2_4 \rightarrow 10000$

$$24 - 1 \rightarrow (1111)_2$$

- Thus 1's complement can be obtained as $(2^n - 1) - N$ or subtracting each digit of number from 1.

Example 9: 1's complement of 1011000

Solution: $1111111 - 1011000 = 0100111$

Note: It is similar to changing 1's to 0's and 0's to 1 or complement each digit of number is similar to taking 1's complement of the number.

Note: $(r - 1)$'s complement of octal or hexadecimal number is obtained by subtracting each digit from 7 and F respectively.

Example 10: Obtain 15's complement of number $(3241)_{16}$

Solution: Subtracting each digit of number from FFFF:

$$\begin{array}{r} \text{F F F F} \\ - 3 2 4 1 \\ \hline \text{C D B E} \end{array}$$

15's complement is $(\text{CDBE})_{16}$.

(ii) Radix Complement:

r 's complement of n digit number N in base r is defined as $r^n - N$ for $N \neq 0$ & 0 for $N = 0$

It is equivalent to adding 1 to $(r - 1)$'s complement.

If $(r - 1)$'s complement is given, r 's complement can be obtained.

Example: Find r's complement of 546700 if its 9's complement is 453299.

Solution: r's complement is $453299 + 1$

$$r's \text{ complement} = 453300$$

Example 11: 2's complement of 1010110 is:

Solution: 1's complement: complement each digit of number (1010110) \rightarrow (0101001)₂

Thus 2's complement is $0101001 + 1$

$$2's \text{ complement} = (0101010)_2$$

Another Method to Obtain 10, 2's Complement:

Leaving all least significant 0's unchanged, subtracting the first non-zero least significant digit from 10 and subtracting all higher significant digits from 9.

Example 12: Find 10's complement of 012398

Solution:

1. Subtract 8 from 10 in the least significant position
2. Subtracting all other digits from 9.

$$\begin{array}{r} 9999910 \\ - 012398 \\ \hline 987602 \end{array}$$

Thus 10's complement of 012398 is 987602.

Example: 13 10's complement of 246700.

Solution: Leaving 2 least significant 0's unchanged, subtracting 7 from 10 and other 3 digits from 9.

$$\begin{array}{r} 9991000 \\ - 246700 \\ \hline 753300 \end{array}$$

Thus 10's complement of 246700 is 753300

Similarly 2's complement can be formed by leaving all least significant 0's and first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits.

Example 14: 2's complement of (1101100)₂:

Solution: 1101100

Remain unchanged

Remain unchanged

Reverse all digits

0010100

Thus 2's complement of 1101100 is $(0010100)_2$

Subtraction with complement:

1. Convert subtrahend N to r 's complement.
2. Then add to the minuend M .
3. If $M \geq N$, sum will produce end carry, which can be discarded, what is left is the result, $M - N$.
4. If $M < N$, sum does not produce carry and is equal to $r^n - (N - M)$, which is same as r 's complement of $(N - M)$.
5. To take the answer in familiar form, take the r 's complement of the sum and place a negative sign in front.

Example 15: Using 10's complement, subtract $72532 - 3250$

Solution:

$$M = 72532$$

$$N = 03250$$

10's complement of $N = 96750$

$$\begin{array}{r} \text{Sum:} \quad 72532 \\ \quad + 96750 \\ \hline 169282 \end{array}$$

Discard end carry as $M > N$ so result: 69282

Example 16: Using 10's complement, subtract $3250 - 72532$

Solution: $M = 3250$

$$N = 72532$$

10's complement of 72532 is

$$\begin{array}{r} 9999 \ 10 \\ - 7253 \ 2 \\ \hline \end{array}$$

10's complement 27468

$$\begin{array}{r} \text{Sum:} \quad 3250 \\ \quad 27468 \\ \hline \end{array}$$

Sum 30718

Since $N > M$ so no end carry.

Therefore answer is $-(10\text{'s complement of } 30718) = -69282$

Example 16: Subtract $1010100 - 1000011$

Solution: 2's complement of N (1000011)= 0111101

$$\begin{array}{r} \text{Sum:} \quad 1010100 \\ \quad \quad + 0111101 \\ \hline \quad \quad 10010001 \end{array}$$

So result is 0010001

Example 17: Subtract: $1000011 - 1010100$

Solution: 2's complement of $1010100 \rightarrow 0101100$

$$\begin{array}{r} \text{Sum:} \quad 1000011 \\ \quad \quad + 0101100 \\ \hline \quad \quad 1110111 \end{array}$$

There is no end carry. Therefore, answer is $-(2\text{'s complement of } 1110111)$

$$= -0010001$$

Note: Subtraction can also be done using $(r-1)$'s complement.

Signed Binary numbers: When binary number is signed, left most bit represents the sign and rest of bits represent the number.

- If binary number is unsigned, then left most bits is the most significant bit of the number.
- Positive or Negative can be represented by (0 or 1) bit which indicate the sign.

Example 19: String of bits 01001 can be considered as 9 (unsigned binary) or +9 (signed binary) because left most bits are 0.

Example 20: String of bits 11001 represent 25 when considered as unsigned number or -9 when considered as signed number.

Negative number representation:

- (i) Signed magnitude representation: In this representation number consist of a magnitude and a symbol (+ or -) or bit (0 or 1) indicating the sign. left most bit represents sign of a number.

E. : 11001 \rightarrow -9

01001 \rightarrow +9

(ii) Signed complement system:

- In this system, negative number is indicated by its complement.
- It can use either 1's or 2's complement, but 2's complement is most common.

Note:

1. 2's complement of positive number remain number itself.
2. In both signed magnitude & signed complement representation, the left most significant bit of negative numbers is always 1.

Example: +9 @ 00001001

-9 @ 11110111 (2's complement of +9)

Note: Signed complement of number can be obtained by taking 2's complement of positive number including the sign bit.

- Signed magnitude system is used in ordinary arithmetic, can not employed in computer arithmetic because of separate handling of the sign and the magnitude.
- In computer arithmetic signed complement system is used to represent negative numbers.

Decimal	Signed 2'Complement	Signed 1's complement	Signed magnitude
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	-	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100

Arithmetic addition:

- Addition in signed magnitude system follows rules of ordinary arithmetic.
- EX. : +25 + - 37 = -37 + 25 = -12

- Thus In this, comparison of sign and magnitude and then performing either addition or subtraction.
- But in signed complement system, only addition, it does not require comparison & subtraction.
- In signed complement system, negative numbers are represents in 2's complement form and then addition to other number including their sign bits.

Example:

+ 6	00000110	- 6	11111010 (2's complement)
<u>+13</u>	00001101	+13	00001101
+19	00010011	+7	10000011
+ 6	00000110	- 6	11111010
- 13	11110011	-13	11110011
- 7	11111001	-19	11101011

[Left significant bit is 1 so number is negative, number will be $-(2's \text{ complement of } 11111001)$

$$= -(00000111) = -7$$

Number will be: $-(2's \text{ complement of } 11101011) = -(00010101) = -(19)$

Note: If result of sum is negative, then it is in 2's complement form.

The left most significant bit of negative numbers is always 1.

- If we use signed complement system, computer needs only one hardware circuit to handle both arithmetic (signed & unsigned), so generally signed complement system is used.

Binary Codes:

Any discrete element of information distinct among a group of quantities can be represented with a binary code.

- n bit binary code is a group of n bits that have 2^n distinct combinations of 1's and 0's with each combination representing one element of the set that is being coded.

Example: With 2 bits $2^2 = 4$ elements can be coded as: 00, 01, 10, 11

With 3 bits $2^3 = 8$ elements can be coded as:

000, 001, 010, 011, 100, 101, 110, 111

- Minimum number of bits required to code 2^n distinct quantities in n.
- The bit combination of an n bit code is determined from the count in binary from 0 to $2^n - 1$.

Example: 3 bit combination

000 0

001 1

010 2

011 3

100 4

101 5

110 6

111 7

BCD code:

Binary coded decimal

- A number with k decimal digits require 4 K bits in BCD.
- A decimal number in BCD is same as its equivalent binary number only when number is between 0 to 9.
- BCD number needs more bits than its equivalent binary.
- Example: $(185)_{10} = (000110000101)_{BCD} = (1011101)_2$
- In BCD number, each bit is represented by its equivalent binary representation.

Note: BCD numbers are decimal numbers and not binary numbers, although they use bits in their representation.

- Decimal are written as 0,1,2,3, ..., 9 which BCD can be written as : 0000, 0001, 0010, 0011, ..., 1001

Benefits of BCD:-

- BCD helps to do arithmetic operation directly on decimal numbers without converting them into equivalent binary numbers.

Decimal system	BCD digits	Binary equivalent
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110

7	0111	0111
8	1000	1000
9	1001	1001
10	00010000	1010
11	00010001	1011

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