SAMPLE STUDY MATERIAL

Postal Correspondence Course

GATE, IES & PSUs

Civil Engineering

FLUID MECHANICS & OCF

28-B/7, Jia Sarai, Near IIT, Hauz Khas, New Delhi-110016. Ph. 011-26514888. www.engineersinstitute.com

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CONTENT

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CHAPTER-1

PROPERTIES OF FLUID

Fluid: A substance capable of flowing and which will undergo a deformation on application of a force shear and deformation continues as long as the force exists. e.g. liquid, gas.

(1) Ideal fluid:

- Non-viscous, incompressible
- Surface tension doesn't exist
- Offers no resistance against flow
- Also known as **imaginary fluid**
- For mathematical analysis, fluids with low viscosity are treated as ideal fluid.

  e.g. Air, water etc.

(2) Real fluid :

- Viscous, compressible
- Surface tension exists
- Offers resistance against fluid
- Mostly fluid available in nature are real fluid

Properties of Fluid:

(1) Density/mass density, ° : Defined as the ratio of the mass of a fluid to its volume.

$$\rho = \frac{M}{v}$$

Where, ρ : density (kg/m^3)

M : mass (kg)

v : volume (m^3)

- ρ_w(water) = 1000 kg/m^3 or 1g/cm^3

  ' ° ' depends on temperature & Pressure.

Temperature↑ ⇒ °↓
Pressure↑ ⇒ °↑

(2) Specific weight, ¥: Defined as the ratio of the weight of fluid to its volume.

$$\gamma = \frac{\text{weight}}{\text{volume}} = \frac{mg}{v}$$

$$\gamma = \rho g$$

Where, γ : specific wt. (N/m^3)

m : mass (kg)

g : acceleration due to gravity = 9.81 m/s^2

v : volume (m^3)

- **$¥_w$(water) :** 9.81 kN/m^3

(3) Specific volume ³ **:** Defined as volume per unit mass of a fluid.

Where, $V \rightarrow$ volume

$M \rightarrow$ Mass

$$\vartheta = \frac{v}{M} = \frac{1}{\rho}$$

$\rightarrow$ Reciprocal of density

$\rightarrow$ Unit $\rightarrow m^3/kg$

- **Commonly used for gases.**

(4) Specific gravity (G) : Defined as the ratio of specific weight (weight density) of a fluid to the specific weight of a standard fluid.

- For liquid, **water** is taken as standard fluid.
- For gas, **air** is taken as standard fluid.
- For liquid, $G = \frac{\gamma_l}{\gamma_w}$

Where,

γ_l = specific weight of liquid

γ_w = specific weight of water

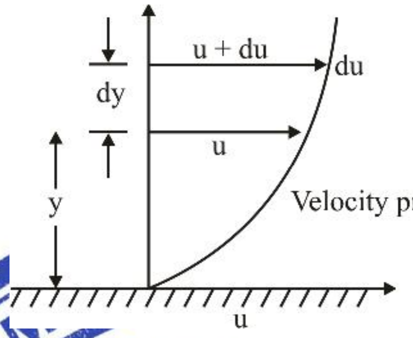
- For gas, $G = \frac{\gamma_g}{\gamma_{air}}$

  γ_g = specific weight of gas

  γ_{air} = specific weight of air

- **For, mercury, G = 13.6**

Viscosity : It is that property of a fluid by virtue of which it offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.



Hence, mathematically,

$$\Gamma \propto \frac{du}{dy}$$

$$\Rightarrow \quad \Gamma = \mu \frac{du}{dy}$$

Where, Γ = shear stress (N/m^2)

$\frac{du}{dy}$ = rate of shear strain (1/s) OR rate of shear deformation OR velocity gradient

μ : Co-efficient of dynamic viscosity/viscosity or viscosity of fluid

Equation $\Gamma = \mu . \frac{du}{dy}$ is called as Newton's equation of viscosity.

- **Unit of « :**

In SI unit, unit of μ = $N.s/m^2$ or Pa.s $\left[\because 1N/m^2 = 1Pa\right]$

In CGS unit, unit of $\mu = \frac{dyne-sec}{cm^2} = 1 poise$

In MKS unit, unit of $\mu = \dfrac{kgf - sec}{m^2}$

- Unit is larger than the number. Write in proper manner i.e. $1\dfrac{N-S}{m^2} = 10\,\text{poise}$
- 1 Centipoise (=1 cp) = $\dfrac{1}{100}\,\text{poise}(=p)$
- **Viscosity of water** (μ) at 20°c = 0.01 poise or 1 cp.

Kinematic viscosity, υ : Defined as the ratio of co-efficient of dynamic viscosity (μ) to the density (ρ) of fluid.

$$\boxed{\upsilon = \frac{\mu}{\rho}}$$

Unit of ^{3} **:**

- In SI unit, unit of υ : m^2/s
- In CGS unit, unit of υ : cm^2/sec = stoke
- 1 Stoke = $10^{-4}\ m^2/s$ & 1 Centistokes = 1/100 stoke

Variation of viscosity with temperature:

Liquid : $$\boxed{\mu = \mu_0 \left\{\frac{1}{1+\alpha t+\beta t^2}\right\}}$$

μ : Viscosity of liquid at t°c (in poise)

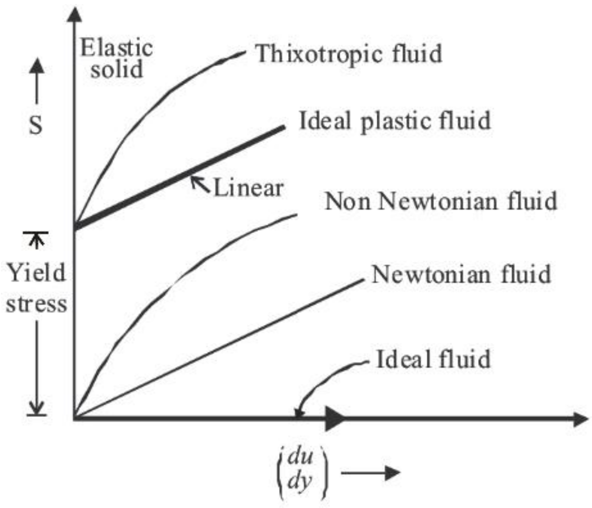
μ_0 : Viscosity of liquid at 0°C (in poise)

α, β : Constants

- With increase in temperature, viscosity decreases.
- Here, cohesive forces predominate, which get reduced with increase in temperature.

Gas : $$\boxed{\mu = \mu_0 + \alpha t - \beta t^2}$$

- With increase in temperature, viscosity increases.
- Here, molecular momentum transfer predominates which increases with increase in temperature.



Types of fluids:

- **Newtonian fluid :** These obey Newton's law of viscosity i.e., $\boxed{\Gamma = \mu \frac{du}{dy}}$

  e.g: air, water, Glycerin, kerosene etc.

- **Non-Newtonian Fluid:** These donot obey newton's law of viscosity i.e. $\boxed{\Gamma \neq \mu \frac{du}{dy}}$

- **Ideal plastic:** It has a definite yield stress and a linear relation exists between shear stress (Γ) and rate of shear strain $\frac{du}{dy}$.

- **Thixotropic fluid:** It has a definite yield stress and a non-linear relation exists between shear stress (Γ) and rate of shear strain (du/dy)   e.g. Printer's ink

- **The study of non-Newtonian fluid is termed as 'rheology'**

Non Newtonian fluid further classified as

$$S = P\left(\frac{du}{dy}\right)^n + q$$

Case I: Dilatant Fluid

For $q = 0; n > 1$

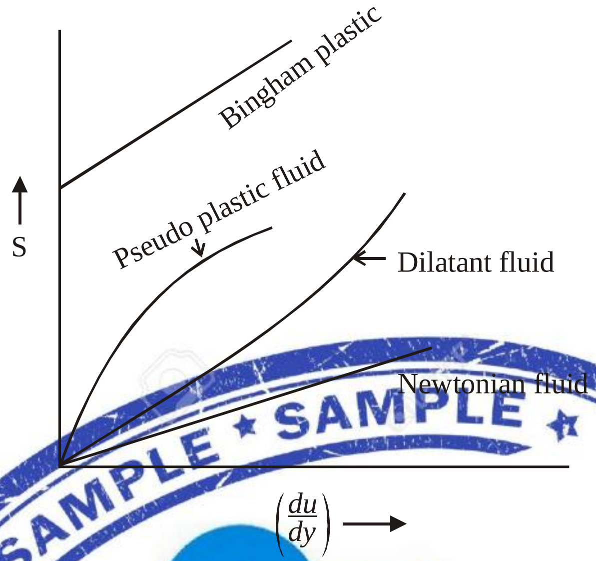
$$S = P\left(\frac{du}{dy}\right)^n = P\left(\frac{du}{dy}\right)^{n-1} . \left(\frac{dy}{dy}\right)$$

$$\boxed{S = \mu_{APR}\left(\frac{du}{dy}\right)}$$

In this case μ_{APR} (Apparent viscosity) increases as the rate of shear deformation increases. So these fluid called shear thickening fluids.

E.g. : ⇒ Sugar in water

⇒ Butter



Case-II: Pseudo-plastic Fluid $(q = 0 \text{ and } n < 1)$

$$S = A\left(\frac{du}{dy}\right)^n = A\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy}$$

$$\boxed{S = \mu_{APR}\left(\frac{du}{dy}\right)}$$

μ_{APR} (Apparent viscosity) decreases as the rate of shear deformation Increases so these fluid called shear thining fluids. E.g. : Blood, Milk etc

Case III. Bingham Plastic $(q \neq 0 \;\; n = 1)$

$$\boxed{S = P\left(\frac{du}{dy}\right) + q}$$

Example: Tooth Paste

<u>Bulk modulus, K</u> : Defined as the ratio of compressive stress to volumetric strain.

$$k = \frac{dp}{\left(-\frac{dv}{v}\right)}$$

Where, dp : change in pressure = compressive stress.

$\frac{dv}{v}$: volumetric strain = $\frac{\text{change in volume}}{\text{original volume}}$

- **k (for water) at normal temperature & pressure =** 2.06×10^9 N/m^2
- **k (for air) at normal temperature & pressure =** 1.03×10^5 N/m^2
- Hence, air is about 20,000 times more compressible than water.
- It is temperature dependent.

Compressibility : It is given as the reciprocal of bulk modulus (k).

Compressibility = 1/k

Where, $K = \frac{dP}{-\frac{dV}{V}}$

but $-\frac{dV}{V} = \frac{d\rho}{\rho}$ $\quad K = \frac{\rho dP}{d\rho}$

$$\beta = \frac{1}{K} = \frac{d\rho}{\rho dP}$$

Where, $\beta \rightarrow$ Compressibility of fluid

$K \rightarrow$ Bulk modulus

$\rho \rightarrow$ Density

$P \rightarrow$ Pressure intensity

Surface tension:

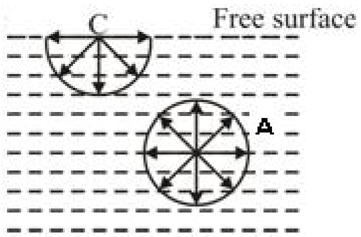
- The property of the liquid surface film to exert a tensile force in called surface tension.
- It is given by tensile force required to maintain unit length of film in equilibrium.

i.e., $S = \frac{T}{l}$

Where, S : surface tension (N/m)

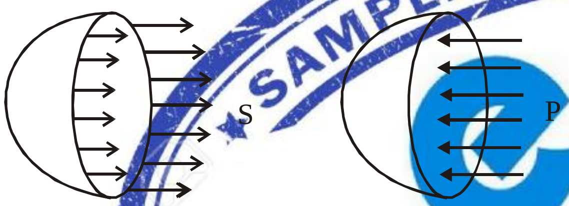
T : tensile force (N)

l : Length (m)



- It is temperature dependent and decreases with rise in temperature.
- It is also dependent on the fluid in contact with the liquid surface.

Case I: Pressure intensity inside a liquid a droplet.



$$p = \frac{2S}{r}$$

Where, p : pressure intensity inside the droplet.

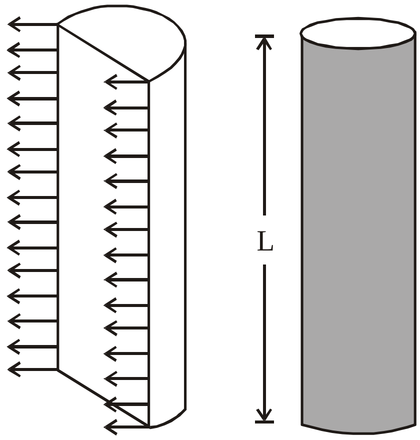
S : surface tension.

r : radius of droplet.

Case II: Pressure intensity inside a soap bubble.

$$p = \frac{4S}{r}$$

Case III: Surface tension on a liquid jet consider a jet of length l, and diameter d.



Force due to pressure = P × L × d

Force due to surface tension = S × 2L

For equilibrium   P × L × d = S × 2L $P = \frac{S \times 2L}{L \times d} = \frac{2S}{d}$

Where,   P – Pressure intensity inside the jet (N/m^2)

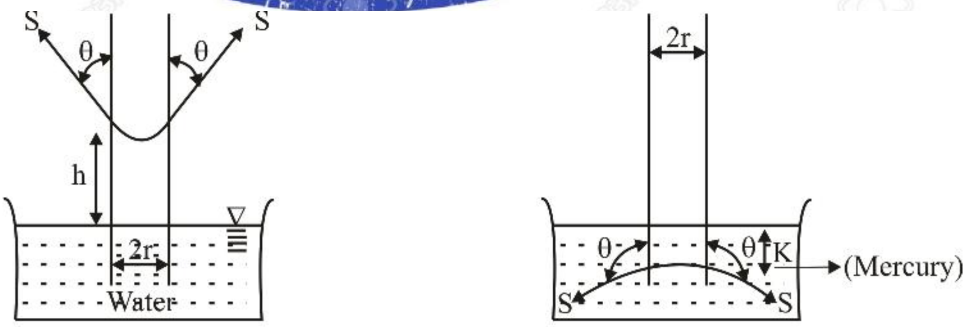
S – Surface tension (N/m)

L – Length of jet (Assumed)

d – Diameter of Jet

Here, two surfaces are in contact with air

Capillarity: A phenomenon of rise or fall of liquid surface relative to adjacent general level of liquid is known as capillarity.



Capillary rise   Capillary depression

- Capillary, rise, $h = \frac{2S\cos\theta}{r\rho g}$

Where,   h : capillary rise or fall (m).

θ : Angle of contact between liquid and glass tube.

ρ : density of liquid (kg/m^3)

g : 9.81 m/s^2 (Acceleration due to gravity)

S : surface tension (N/m)

- **For water and clean glass tube, θ = 0º**
- **For mercury and glass tube, θ = 128º**
- Above equation of capillary rise holds true for small radius, $r < 2.5$ mm.
- For radius of tube $r \geq 6$ mm, value of 'h' becomes negligible.

Special case :

If the tube of radius, r is inserted in a mercury of specific gravity (S_1) and above which another liquid of specific gravity (S_2) lies then capillary depression,

$$h = \frac{2S\cos\theta}{r\gamma_w(S_1 - S_2)}$$

γ_w : specific weight of water.

Vapour pressure :

If the vapour molecules (Generated due to vaporization) are confined in a closed vessel, these vapour molecules get accumulated in the space between the free liquid surface and the top of vessel. Hence, the partial pressure exerted on the liquid surface due to this accumulated vapour is known as vapour pressure of the liquid.

OR

It can be said that this is the pressure at which the liquid is converted into vapour.

- A liquid may boil at ordinary temperature (other than boiling temperature). If the pressure above the liquid surface is reduced to be equal or less than the vapour pressure of the liquid at that temperature.

Cavitation : A phenomenon of formation of vapour bubbles in a flowing liquid at a region where the pressure of liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in the region leads to higher pressure. This high pressure causes cavity formation in adjoining areas.

Question & Solution

Example-1: At a certain point in castor oil the shear stress is 0.216 N/m^2 and velocity gradient 0.216 sec^{-1}. If the mass density of castor oil is 959.42 kg/m^3 find kinematic viscosity

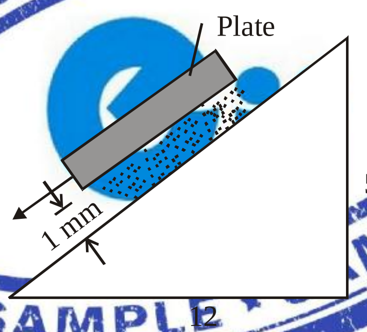
Solution: Kinematic viscosity $\upsilon = \frac{\text{Dynamatic Viscosity}(\mu)}{\text{Density }(\rho)}$

$$S = \mu\left(\frac{du}{dy}\right) \qquad 0.216 = \mu(0.216) \quad \mu = 1\,N.S/m^2$$

$$3 = \frac{1}{959.42} = 1.042\times10^{-3} m^2 / sec$$

Example 2: A square plate of size 1m × 1m and weighting 350 N slides down an inclined plane with a uniform velocity of 1.5 m/sec. The inclined plane is laid on a slope of 5 vertical to 12 horizontal. Oil film thickness = 1mm

$\mu = ?$



Solution: $F = W\sin\theta = 350\times\frac{5}{13} = 134.615$

$$S = \mu\left(\frac{du}{dy}\right) \qquad du = u - 0 = 1.5 \text{ m/sec}$$

$$\frac{F}{A} = \mu\frac{du}{dy} \qquad \mu = \left(\frac{134.615}{1}\right)\times\frac{1\times10^{-3}}{1.5}$$

$$\boxed{\mu = 0.897 \text{ Poise}}$$